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Unit- 1.Electric charges & field

Imagine
with all
your mind.
Believe
with all
your heart.
Achieve
with all
your might.

UNIT-1

ELECTRIC CHARGES AND FIELD

ELECTRIC CHARGE

The property of material by which it exerts or experience electric and magnetic effects is called the electric charge.

Examples-

1. Experience of seeing a spark or hearing a crackle, when we take of our synthetic cloths.
2. Lightning in the sky during thunderstorms due to electric discharge.

ELECTROSTATICS

The branch of Physics, which deals with the study of charges at rest (static charges), the forces, fields and potential due to these charges is called Electrostatics or static Electricity.

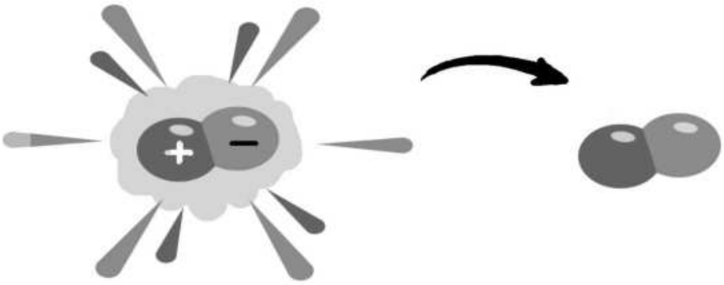
Important Facts about charge

(1.) There are only two types of electric charge.

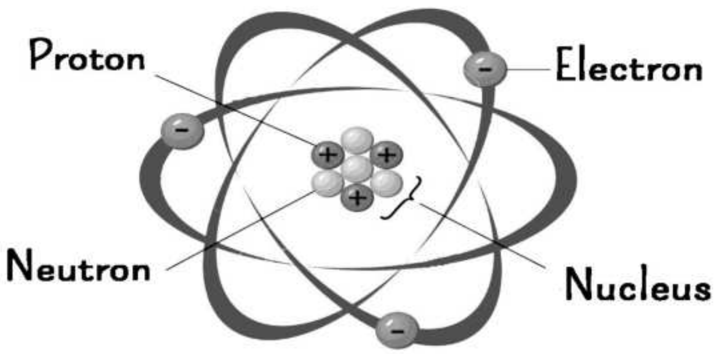
(2.) Like charges repel and unlike charges attracts each other.

(3.) The unlike charges nullify (cancel) each other's effect when they come in contact. Therefore the charges are named as positive and negative by the American scientist 'Benjamin Frenklin'.

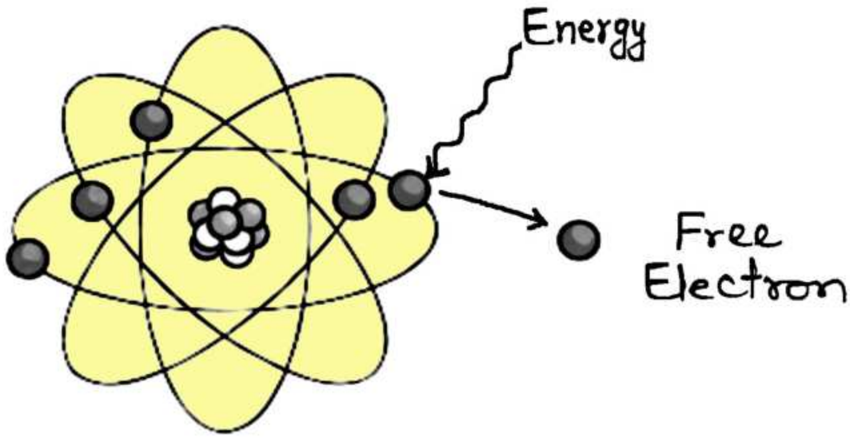
Benjamin Frenklin



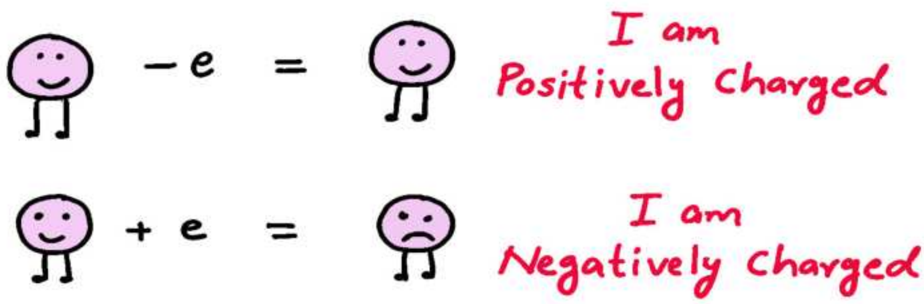
(4.) Normally the materials are electrically neutral, they do not contain charge because their charges (Protons and electrons) are exactly balanced.



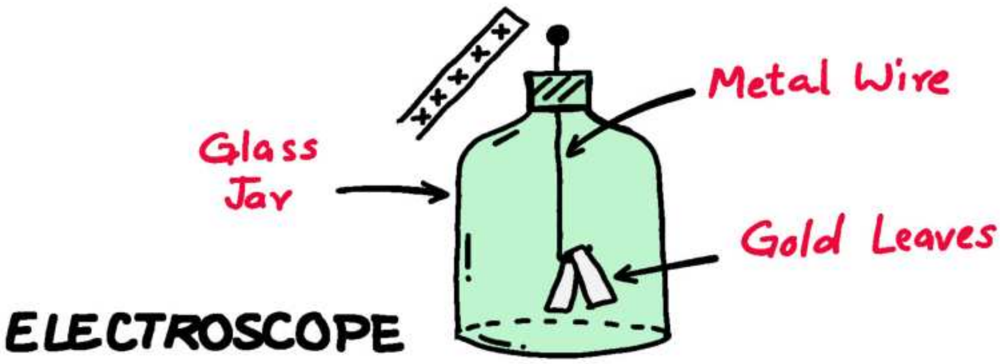
(5.) The electron of the outermost orbit of an atom are far from the nucleus so these electrons are Loosely bounded with the nucleus and can be separated easily from the orbit by giving some energy. These electron are then called free electrons.



(6.) The body or atom can be charged positively by Loosing some of its electrons and can be charged negatively by gaining electrons.



(7.) The apparatus used to detect charge on a body is "Gold leaf Electroscope".



(8.) An electric charge can creates electric field (E), magnetic field ($\vec{B}$) and electromagnetic radiations

V = 0  Only Electric Field

 Electric Field + Magnetic Field

V = Constant

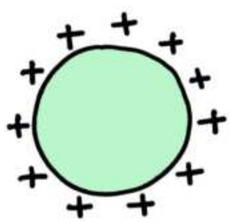
 Electric Field + Magnetic Field + EM Waves

V ≠ Constant

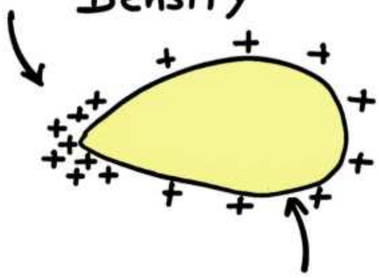
(9.) Electric charge uniformly distribute on a uniform surface but the charge density on a non-uniform surface is maximum on that points where the radius of curve is minimum.

Charge density $\sigma \propto \frac{1}{R}$

More Surface Charge Density



Uniform Surface Charge Density



Less Surface Charge density

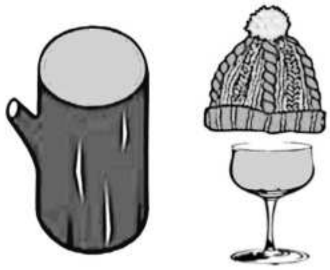
Conductor, Insulator & Semiconductor

Conductor

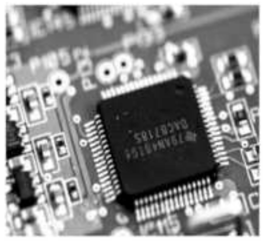
The material through which the electric charge and current can flow easily are called Conductors.

Examples - All Metals, Human body, earth etc.

Conductor



Insulator



Semiconductor

Insulator

The materials which resist the flow of electric charge and current are called insulators.

Examples - Wood, Porcelene, plastic etc.

Semiconductors

The materials which resist the flow of electric charge and current but the value of their resistance is in between the conductor and insulator are called semiconductors.

Examples - Silicon (Si), Germenium (Ge)
Carbon (C) → Excellent thermal Conductivity.

Sharpen Your Pencil

Charge is the property associated with matter due to which it produces and experiences

(a) electric effects only

(b) magnetic effects only

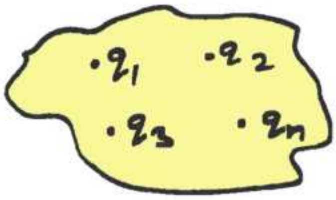
(c) both electric and magnetic effects

(d) None of these

Properties of Electric Charge

(1) Additivity of Charges

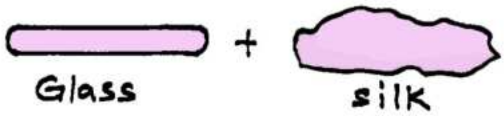
Electric charge is a scalar quantity so the total charge of the system is obtained simply by adding the different charges algebrically with proper sign according to their charge.



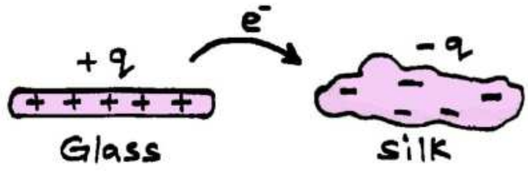
$$q = q_1 + q_2 + \dots\ q_n$$

(2) Charge is conserved

Charge can not be either created or distroyed. It can only be transferred from one body to another. Hence the total charge of the isolated system is always conserved.



Total charge = 0+0 = 0



Total charge = +q − q = 0

(3) Quantisation of charge

The electric charge on a body is always an integral multiple of e (charge on 1 electron). This is called quantisation of charge.

$$\boxed{Q = \pm ne}$$

$n = 0, 1, 2 \dots \infty$

$e = 1.602 \times 10^{-19}\ C$

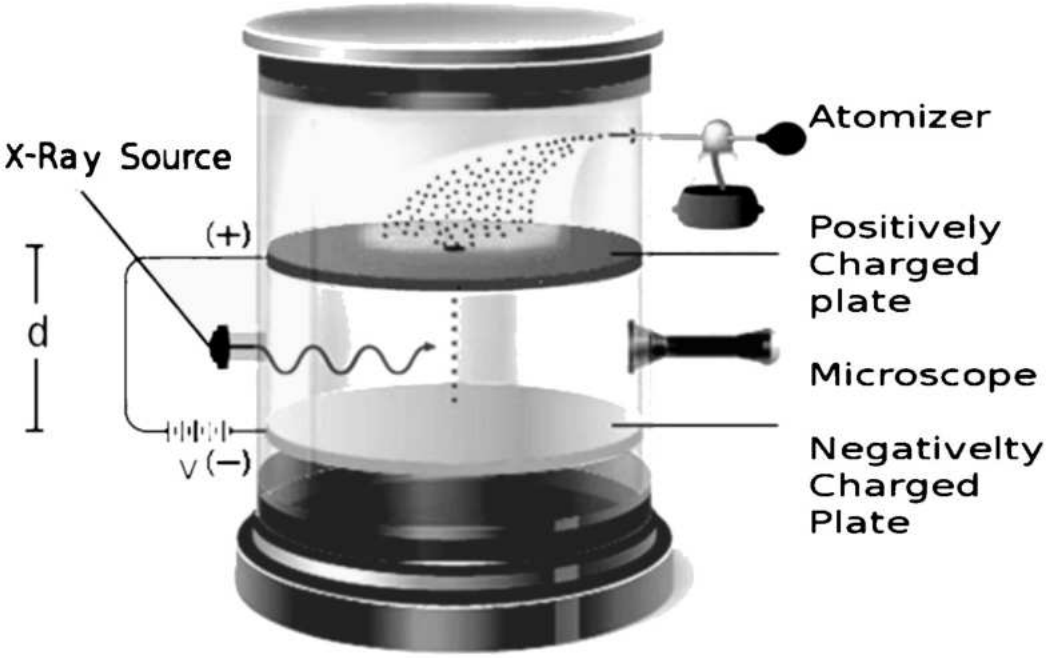
* The unit of electric charge is coulomb.

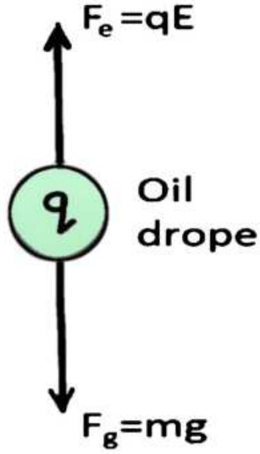
1 Coulomb -

The total charge on 6.24×10^{18} electrons is equal to 1 coulomb.

Millikan's Oil drop Experiment

The charge on one electron was found by this oil drop experiment by Millikan in 1912.





When the oil drop remain stationary in the experiment then the forces on oil drop

$$\text{Electric Force } (F_e) = \text{Gravitational Force } (F_g)$$

$$qE = mg$$

or

$$\boxed{q = \frac{mg}{E}}$$

We know that - $\quad$ mass = density × volume

$$\boxed{m = d \times \frac{4}{3}\pi r^3}$$

where $r \rightarrow$ radius of oil drop (0.5 mm)

* Mass of Electron (m_e) = $9.11 \times 10^{-}$ kg.
* Mass of Proton (m_p) = 1.67×10^{-27} kg.

Q. An oil drop of 12 excess electrons is held stationary under a constant electric field of 2.55×10^4 N/c in Millikan's oil drop experiment. The density of the oil is 1.26 gm/ cm^3. Then estimate the radius of the oil drop.

Sol. Given that -

$$\text{Charge } q = ne = 12 \times 1.6 \times 10^{-19} \text{ C}$$

$$\text{Density } d = 1.26 \times \frac{10^{-3}}{10^{-6}} = 1.26 \times 10^3 \text{ Kg/m}^3$$

when the oil drop is held stationary then

$$q = \frac{mg}{E}$$

We Know $m = d \times \frac{4}{3}\pi r^3$

so that $q = \dfrac{d \times \frac{4}{3}\pi r^3 \times g}{E}$

or $r^3 = \dfrac{qE}{d \times \frac{4}{3}\pi \times g}$

$$r^3 = \frac{12 \times 1.6 \times 10^{-19} \times 2.55 \times 10^4}{1.26 \times 10^3 \times \frac{4}{3} \times 3.14 \times 9.8} = 9.47 \times 10^{-18} \text{ m}^3$$

$$r = (9.47 \times 10^{-18})^{1/3} = 9.81 \times 10^{-7} \text{ m.}$$

Q. What is the total charge on 75 Kg of electrons?

Sol. Number of electrons in 75 Kg.

$$n = \frac{\text{total Mass}}{\text{mass of 1 electron}} = \frac{75}{9 \times 10^{-31}}$$

$$n = 8.3 \times 10^{31}$$

so Total charge $Q = ne$

$$Q = 8.3 \times 10^{31} \times (-1.6 \times 10^{-19})$$

$$Q = -1.33 \times 10^{13} \text{ C}$$

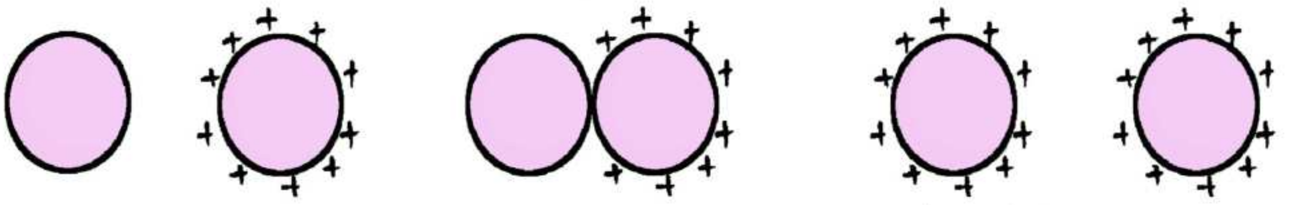
Methods of Charging

1.) Charging by Friction

when two objects are rubbed against each other then electrons from the atoms of one object (having low work function) go to other object (having high work function). The material looses electrons gets positively charged and the material gains electrons gets negatively charged.

2.) Charging by conduction

when a charged object is brought in contact with an uncharged conductor, then the same type of charge spreads on both the conductors. This happens because some electrons are transferred at the contact point.

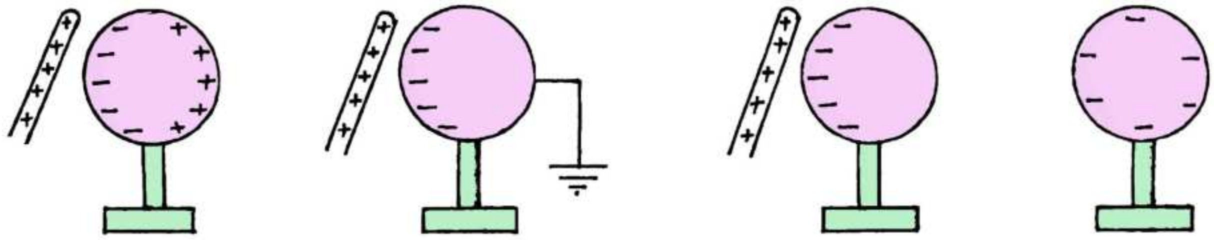


3.) Charging by Induction

Method 1. (Using a ground Connection)

(i) A positively charged rod is brought near a neutral metal sphare and polarising it.

(ii) The sphare is grounded allowing electrons to be attracted from the earth.



(a) (b) (c) (d)

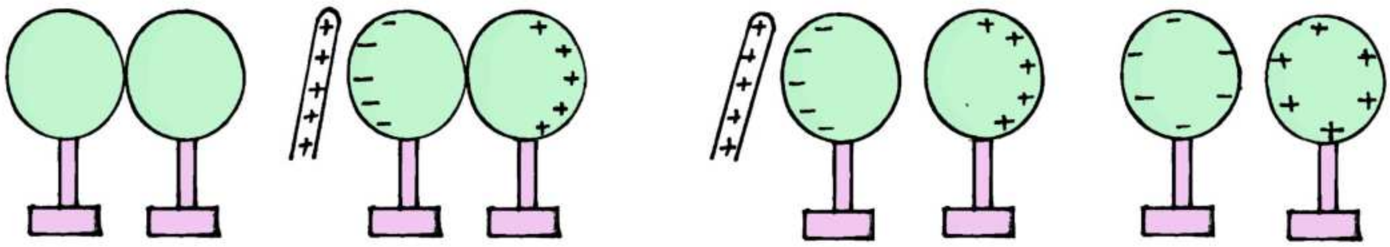
(iii) The ground connection is broken.

(iv) The positive rod is removed leaving the sphare with an induced negative charge.

Method 2.

(i) Two uncharged metal sphares are in contact with each other but insulated from rest of the world.

(ii) A positively charged rod is brought near the sphare, attracting negative charge and leaving the other sphare positively charged.



(a) (b) (c) (d)

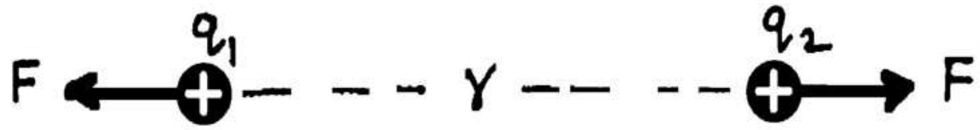
(iii) Now the sphares are separated before the rod is removed and thus separating positive & negative charge.

(iv) Remove the rod. The charges on sphares will rearrange themselves and uniformly distributed over the sphares.

Coulomb's Law

The force of interaction between any two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them, and acts along the line joining the two charges.

Suppose two point charges q_1 and q_2 are separated in vacuum by a distance r.



According to coulomb's Law,

$$F \propto q_1 q_2$$

and $$F \propto \frac{1}{r^2}$$

or $$\boxed{F = \frac{K q_1 q_2}{r^2}}$$

Charles-Augustin de Coulomb

* Electrostatic constant K depends on the nature of medium separating the charges and on the system of units.

In cgs system, $K = 1$

In SI system, $K = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \ Nm^2/C^2$

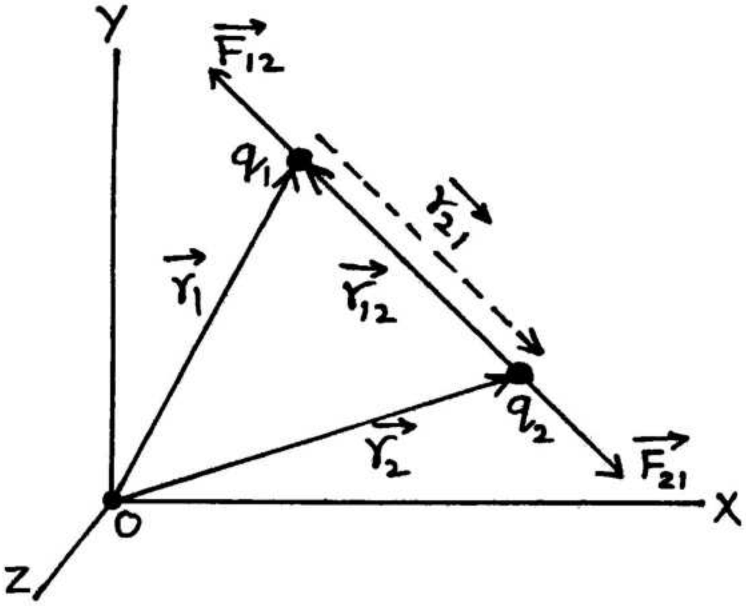
* ϵ_0 electrical permittivity of free space

$$\epsilon_0 = 8.85 \times 10^{-12} \ C^2/Nm^2$$

Coulomb's Law in vector form

Let the position vector of charges q_1 and q_2 be $\vec{r_1}$ and $\vec{r_2}$ respectively.

We denote the force on q_1 due to q_2 by $\vec{F_{12}}$ and the force on q_2 due to q_1 by $\vec{F_{21}}$.



$$\boxed{F_{12} = F_{21}}$$

Here $\vec{r_1} + \vec{r_{21}} = \vec{r_2}$

$$\boxed{\vec{r_{21}} = \vec{r_2} - \vec{r_1}}$$

also $\vec{r_2} + \vec{r_{12}} = \vec{r_1}$

or $\boxed{\vec{r_{12}} = \vec{r_1} - \vec{r_2}}$ and $\boxed{\vec{r_{12}} = -\vec{r_{21}}}$

The magnitude of vectors $\vec{r_{12}}$ and $\vec{r_{21}}$ is denoted by $|\vec{r_{12}}|$ and $|\vec{r_{21}}|$ and their unit vectors are given as-

$$\hat{r}_{12} = \frac{\vec{r_{12}}}{|\vec{r_{12}}|} \quad \text{and} \quad \hat{r}_{21} = \frac{\vec{r_{21}}}{|\vec{r_{21}}|}$$

So the force $\vec{F_{12}}$ and $\vec{F_{21}}$ can be given as-

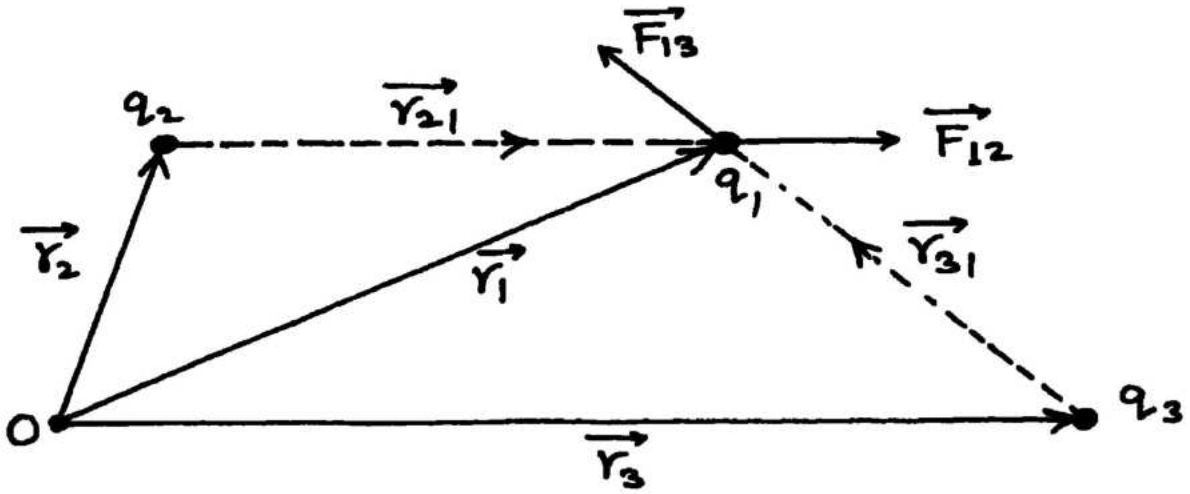
$$\boxed{\vec{F_{12}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r_{12}}|^2}\hat{r}_{12}} \qquad \boxed{\vec{F_{21}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r_{21}}|^2}\hat{r}_{21}}$$

(i) If q_1 and q_2 are of same sign ; $q_1 q_2 > 0$ then $\vec{F_{12}}$ is along $\vec{r_{12}}$, which denotes repulsion

(ii) If q_1 and q_2 are of opposite sign ; $q_1 q_2 < 0$ then $\vec{F_{12}}$ is along $-\vec{r_{12}}$ or $\vec{r_{21}}$, which denotes attraction

Force between Multiple Charges

Total force on any charge due to a number of charges at rest is the vector sum of all the forces on that charge due to other charges.



Let the force on charge q_1 due to two other charges q_2 and q_3 are $\vec{F}_{12}$ and $\vec{F}_{13}$ respectively.

The position vectors of charges q_1, q_2 and q_3 are $\vec{r}_1$, $\vec{r}_2$ and $\vec{r}_3$.

Here $\boxed{\vec{F}_{12} = \frac{Kq_1q_2}{|\vec{r}_{12}|^2}\hat{r}_{12}}$ and $\boxed{\vec{F}_{13} = \frac{Kq_1q_2}{|\vec{r}_{13}|^2}\hat{r}_{13}}$

Thus the resultant force $\vec{F}_1$ on charge q_1 is obtained by the vector sum of the forces $\vec{F}_{12}$ and $\vec{F}_{13}$ as-

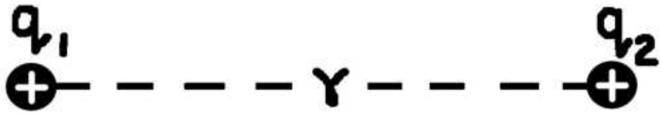
$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13}$$

Similarly if a system have n charges $q_1, q_2, \ldots q_n$ then the force on q_1 due to $q_2, q_3, \ldots q_n$ charges can be calculated by the vector sum of forces $\vec{F}_{12}$, $\vec{F}_{13}$, ... $\vec{F}_{1n}$ as-

$$\boxed{\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \cdots \vec{F}_{1n}}$$

Dielectric Constant (K)

Dielectric constant of a medium is the ratio of absolute electrical permittivity of the medium to the electrical permittivity of free space.



Force in vacuum $(F_0) = \frac{1}{4\pi\epsilon_0} \times \frac{q_1 q_2}{r^2}$ — (1)

Force in a medium $(F_m) = \frac{1}{4\pi\epsilon} \times \frac{q_1 q_2}{r^2}$ — (2)

where ϵ = electrical permittivity of the medium.

Dividing eq. (1) by (2), we get-

$$\boxed{\frac{F_0}{F_m} = \frac{\epsilon}{\epsilon_0} = \epsilon_r \text{ or } K}$$

where ϵ_r or K is called dielectric constant or relative electrical permittivity of the medium.

* Thus force between two given charges in a medium (K) is only $\frac{1}{K}$ times of the force between them in air/ vacuum.

$$\boxed{F_m = \frac{F_0}{K}}$$

* For All Metals $\epsilon_r = \infty$
* For Air $\epsilon_r = 1$

$1 \leq \epsilon_r \leq \infty$

Q. (CBSE 2019,11) Two identical conducting balls A and B have charge $-Q$ and $3Q$ respectively. They are brought in contact and then separated by a distance d apart. Find the nature of Coulomb force between them.

Sol. Final charge on ball A and B $= \frac{3Q - Q}{2} = Q$

So the nature of coulomb force between them is repulsive.

Q. (CBSE 2014) Two equal balls having equal positive charges 'q' are separated by a distance r. What would be the effect on the force when a plastic sheet is inserted between them?

Sol. Force will decreas.

Reason:

Force between two charges in vacuum is

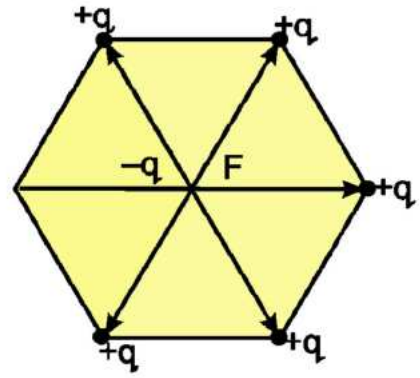
$$F_0 = \frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2}$$

On inserting a plastic sheet (a dielectric $K>1$) then

$$F = \frac{1}{4\pi\epsilon_0 K}\frac{q^2}{r^2}$$

i.e. $$F = \frac{F_0}{K}$$

Q. (CBSE 2019) Five point charges, each of charge $+q$ are placed on five vertices of a regular hexagon of side 'l'. Find the resultant force on a charge $-q$ placed at the centre of the hexagon.

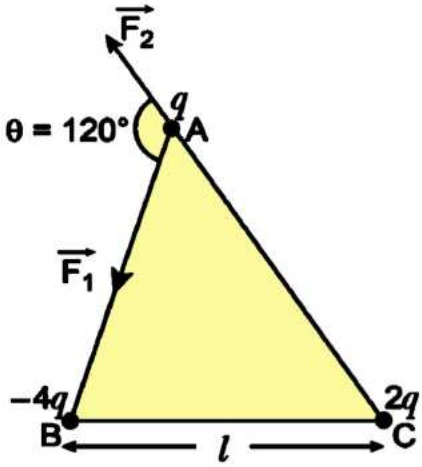


Sol. The force due to the charges placed diagonally opposite at the vertices of hexagon, on the charge $-q$ cancel in pair.

Hence net force is due to one charge only.

$$F = \frac{1}{4\pi\epsilon_0}\frac{q^2}{l^2}$$

Q. (CBSE 2018) Three point charges q, $-4q$ and $2q$ are placed at the vertices of an equilateral triangle ABC of side 'l'. Obtain the expression for the resultant electric force on the charge q.



Sol. Force on charge q due to the charge $-4q$

$$F_1 = \frac{1}{4\pi\epsilon_0}\left(\frac{4q^2}{l^2}\right) \text{ along AB}$$

Force on the charge q due to the charge $2q$

$$F_2 = \frac{1}{4\pi\epsilon_0}\left(\frac{2q^2}{l^2}\right) \text{ along CA}$$

Forces F_1 and F_2 are inclined to each other at an angle $120°$, so the resultant force is

$$F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 120°}$$

$$F = \frac{1}{4\pi\epsilon_0}\frac{q^2}{l^2}\sqrt{16 + 4 - 8}$$

$$F = \frac{1}{4\pi\epsilon_0}\frac{2\sqrt{3}q^2}{l^2}$$

Q. Two point charges of $+2\mu C$ and $+6\mu C$ repel each other with a force of 12 N. Now if each is given an additional charge of $-4\mu C$, what will be the new force?

Sol. Here $q_1 = +2\mu C$, $q_2 = +6\mu C$, $F = 12N$

and $q_1' = +2-4 = -2\mu C$, $q_2' = +6-4 = 2\mu C$

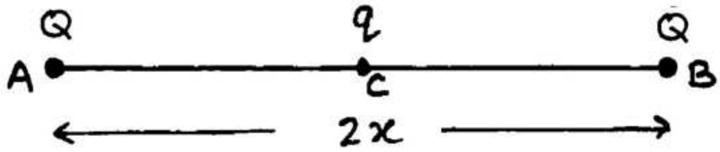
So $$\frac{F'}{F} = \frac{q_1'q_2'}{q_1q_2} = \frac{(-2)(2)}{(2)(6)} = -\frac{1}{3}$$

$$F' = -\frac{F}{3} = \frac{-12}{3} \qquad F' = -4N \text{ (Attractive)}$$

Q. A charge q is placed at the centre of the line joining two equal charges Q. Show that the system of three charges will be in equillibrium if $q = -Q/4$.

Sol.

Let two equal charges Q each, be held at A and B, where $AB = 2x$.



Here C is the centre of AB, where charge q is held. Net force on q is zero. So q is already in equilibrium.

For the three charges to be in equilibrium, net force on each charge must be zero.

Now, total force on Q at B is -

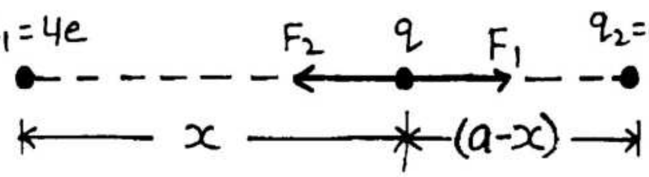
$$\frac{KQq}{x^2} + \frac{KQ \cdot Q}{(2x)^2} = 0$$

$$\frac{KQq}{x^2} = -\frac{KQ^2}{4x^2}$$

So $$\boxed{q = -\frac{Q}{4}}$$ Hence Proved.

Q. Two fixed point charges +4e and +e units are separated by a distance 'a'. Where should the third point charge be placed for it to be in equilibrium.

Sol: For the charge +q to be in equilibrium



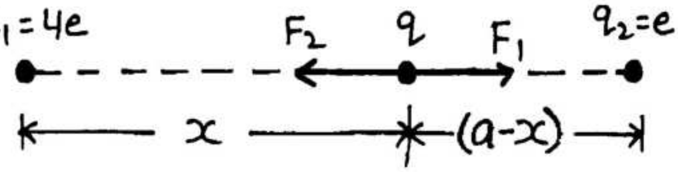
$$F_1 = F_2$$

$$\frac{Kq \cdot 4e}{x^2} = \frac{Kq \cdot e}{(a-x)^2} \Rightarrow \frac{4}{x^2} = \frac{1}{(a-x)^2}$$

on solving - $x = 2a/3$ — Ans.

Q. Two fixed point charges + 4e and +e units are separated by a distance 'a'. where should the third point charge be placed for it to be in equilibrium.

Sol: For the charge +q to be in equilibrium



$$F_1 = F_2$$

$$\frac{Kq \cdot 4e}{x^2} = \frac{Kq \cdot e}{(a-x)^2} \Rightarrow \frac{4}{x^2} = \frac{1}{(a-x)^2}$$

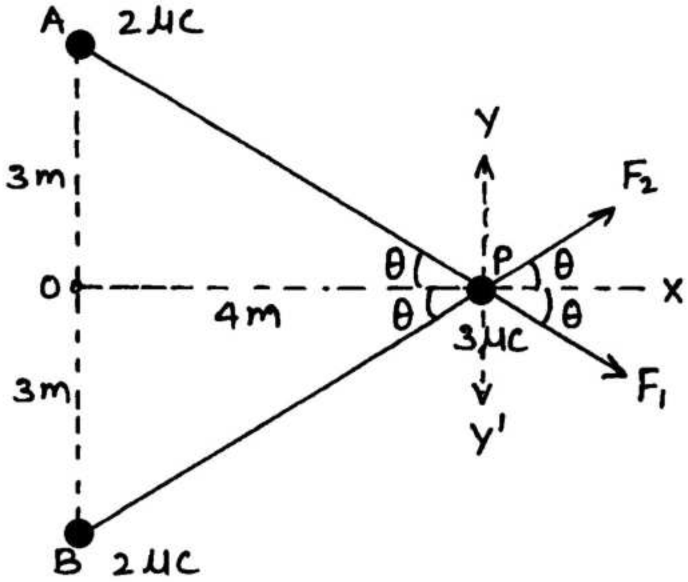
on solving - $x = 2a/3$ — Ans.

Q. Two equal positive charges each of $2\mu C$ interact with a third positive charge of $3\mu C$ situated as shown in fig. Calculate the magnitude and direction of the force on the $3\mu C$ charge.

Sol:

Here $OA = OB = 3m$
and $OP = 4m$

so $AP = BP = \sqrt{3^2 + 4^2}$
$= 5m$.



According to Coulomb's Law

Force on charge at P due to charge at A and B are equal in magnitude so-

$$F_1 = F_2 = \frac{Kq_1 q_2}{r^2} = \frac{9 \times 10^9 \times 2 \times 10^{-6} \times 3 \times 10^{-6}}{5^2}$$

$$\text{so} \quad F_1 = F_2 = 2.16 \times 10^{-3} \text{ N}$$

Here the components of F_1 and F_2 along Y and axis are equal in magnitude ($F_1 \sin\theta = F_2 \sin\theta$) but opposite in direction so they cancel each other

The Components along PX add up so total force on charge 3 μC at P is -

$$F = 2F_1 \cos\theta = 2 \times 2.16 \times 10^{-3} \times \frac{4}{5}$$

$$\text{So} \quad F = 3.5 \times 10^{-3} \text{ N} \quad \text{— Ans.}$$

Q. (CBSE 2014) Two equal balls having equal positive charges 'q' are separated by a distance r. What would be the effect on the force when a plastic sheet is inserted between them ?

Sol. Force will decreas.

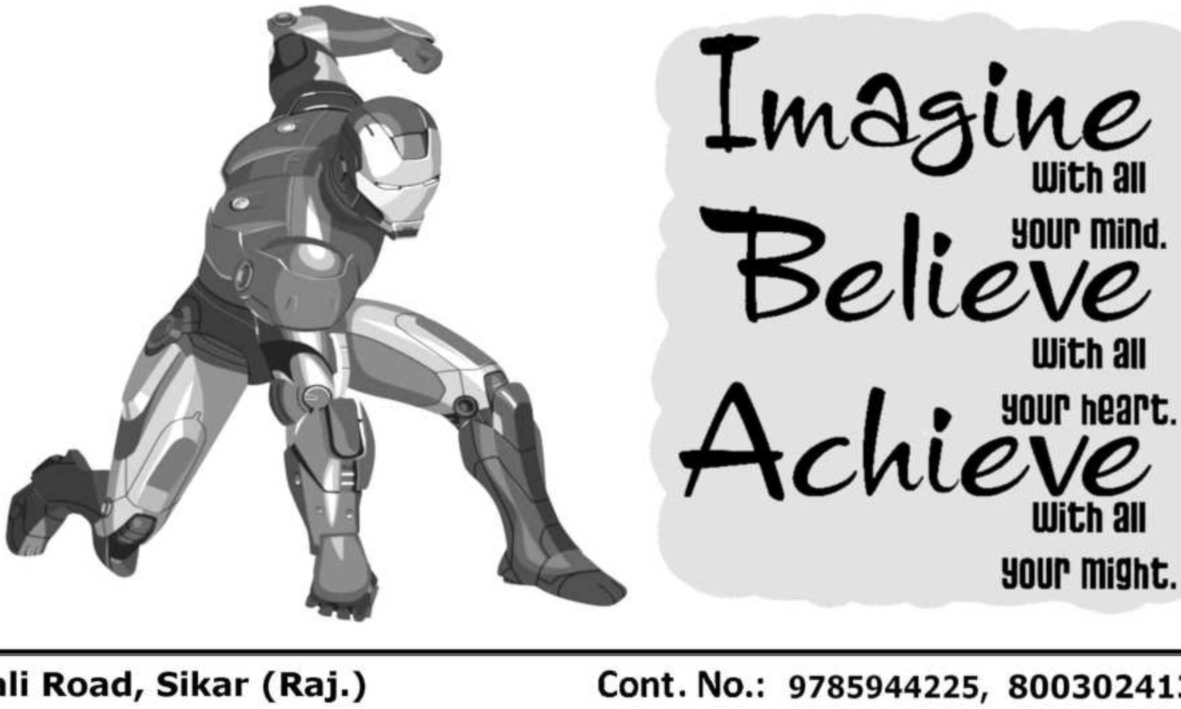
Reason: Force between two charges in vacuum is

$$F_0 = \frac{1}{4\pi\epsilon_0} \frac{q^2}{r^2}$$

On inserting a plastic sheet (a dielectric K>1) then

$$F = \frac{1}{4\pi\epsilon_0 K} \frac{q^2}{r^2}$$

i.e. $$F = \frac{F_0}{K}$$

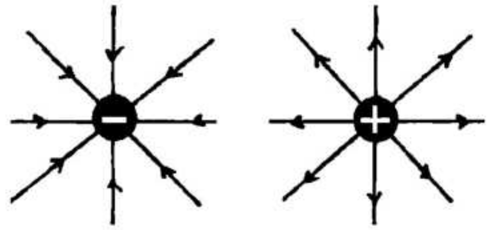


Electric Field

An electric field is the region of space surrounding electrically charged particles where a force is exerted on other electrically charged objects.

* The direction of the field is taken to be the direction of the force it would exert on a positive test charge.

* The electric field is radially outword from a positive charge and radially inward towards a negative charge.



Electric Field Intensity ($\vec{E}$)

The electric field intensity at any point is the strength of electric field at that point. It is defined as the force experienced by unit Positive charge placed at that Point.

According to coulomb's Law force on test charge q_0 due to a point charge Q is

$$F = \frac{1}{4\pi\epsilon_0}\frac{Q \cdot q_0}{r^2}$$

by defintion $\boxed{\vec{E} = \frac{\vec{F}}{q_0}}$ so $\boxed{\vec{E} = \frac{1}{4\pi\epsilon_0}\cdot\frac{Q}{r^2}\,\hat{r}}$

where $\hat{r}$ is unit vector directed from Q towards q_0.

SI Unit of E = $\frac{N}{C}$

Important Points about Electric field

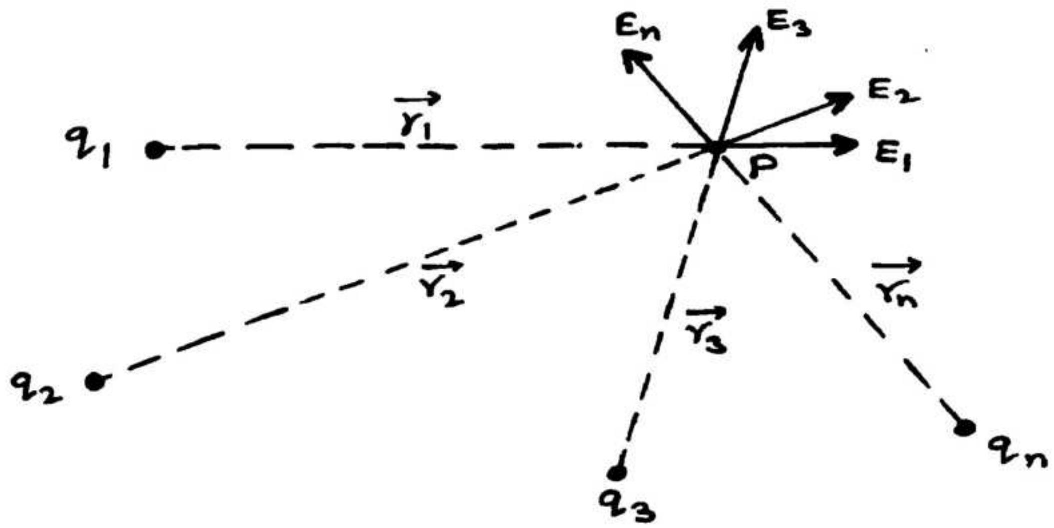
(1) The magnitude of electric field $\vec{E}$ is same at equal distances from the charge Q, thus it has a spherical symmetry.

(2) The test charge q_0 may have its own electric field, it may modify the electric field of the source charge. Therefore to minimize this effect we rewrite electric field intensity at $\vec{r}$ as-

$$\boxed{\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0}}$$

Electric Field due to a system of charges

Consider a system of charges $q_1, q_2, \ldots q_n$ with position vectors $\vec{r}_1, \vec{r}_2, \ldots \vec{r}_n$.

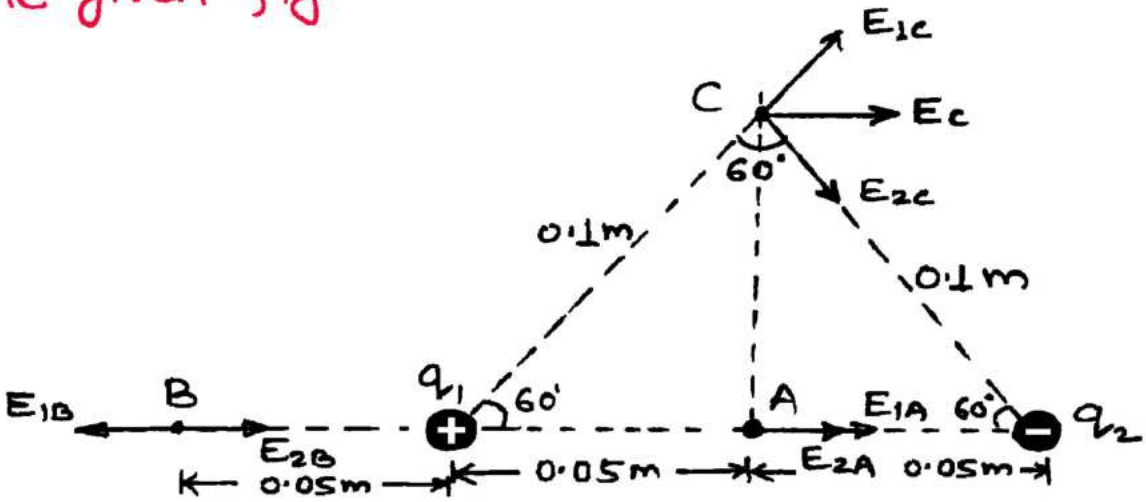


The electric field intensity at a point P due to a system of charges is the vector sum of the electric fields at the point due to individual charges.

$$\boxed{\vec{E} = \vec{E}_1 + \vec{E}_2 + \ldots \vec{E}_n}$$

$$\vec{E} = \frac{Kq_1}{|\vec{r}_1|^2}\hat{r}_1 + \frac{Kq_2}{|\vec{r}_2|^2}\hat{r}_2 + \ldots \frac{Kq_n}{|\vec{r}_n|^2}\hat{r}_n$$

Q. Two point charges q_1 and q_2 of magnitude $10^{-8}C$ and $-10^{-8}C$ respectively are placed 0.1 m apart. Calculate the electric field at points A, B and C in the given fig.



Electric field at point A due to charge q_1

$$E_{1A} = \frac{9\times10^9 \times 10^{-8}}{(0.05)^2} = 3.6\times10^4 \ N/C \ \text{(Towards right)}$$

Electric field at point A due to charge q_2

$$E_{2A} = \frac{9\times10^9 \times 10^{-8}}{(0.05)^2} = 3.6\times10^4 \ N/C \ \text{(Towards right)}$$

Total Electric field at point A $= E_A = E_{1A} + E_{2A}$

$$E_A = 7.2\times10^4 \ N/C \ \text{(Towards the right)}$$

Electric field at point B due to charge q_1

$$E_{1B} = \frac{9\times10^9 \times 10^{-8}}{(0.05)^2} = 3.6\times10^4 \ N/C \ \text{(Towards left)}$$

Electric field at point B due to charge q_2

$$E_{2B} = \frac{9\times10^9 \times 10^{-8}}{(0.15)^2} = 4\times10^3 \ N/C \ \text{(Towards right)}$$

Total Electric field at point B $= E_B = E_{1B} - E_{2B}$

$$E_B = 3.6\times10^4 - 4\times10^3 = 3.2\times10^4 \ N/C \ \text{(Towards Left)}$$

The electric fied at point c due to charge q_1 and q_2 are equal in magnitude

$$E_{1c} = E_{2c} = \frac{9\times10^{9}\times10^{-8}}{(0.1)^2} = 9\times10^{3}\ N/C$$

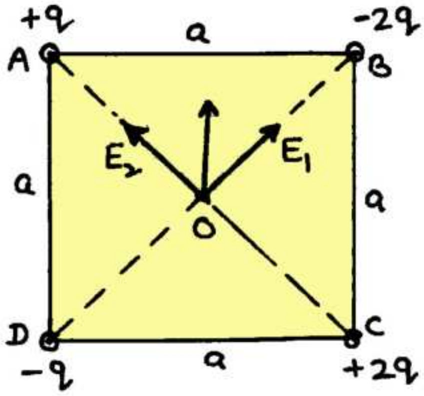
The resultant electric field at point c is

$$E_c = \sqrt{E_{1c}^2 + E_{2c}^2 + 2E_{1c}E_{2c}\cos 120^\circ}$$

$$E_c = \sqrt{E_{1c}^2 + E_{1c}^2 + 2E_{1c}\cdot E_{1c}\times\left(-\frac{1}{2}\right)} \quad [\because E_{1c} = E_{2c}]$$

$$E_c = E_{1c} = 9\times10^{3}\ N/C \quad [\text{Towards right}]$$

Q. What are the magnitude and direction of the electric field at centre of the square in figure. If $q = 10^{-8}C$ and $a = 5$ cm.



Sol. Here

$OA = OB = OC = OD = r$

$$r = \frac{5\times10^{-2}}{\sqrt{2}}\ m.$$

also here $E_1 = E_2 = \dfrac{q}{4\pi\epsilon_0 r^2}$

or $$E_1 = E_2 = \frac{10^{-8}\times9\times10^{9}}{(5\times10^{-2}/\sqrt{2})^2} = 7.2\times10^{4}\ N/C$$

In square ABCD the $\angle AOB = 90^\circ$

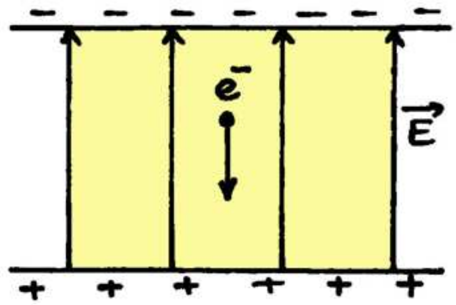
So the Net Electric field at O $= \sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos 90^\circ}$

$$E = \sqrt{E_1^2 + E_2^2} = E_1\sqrt{2}$$

$$E = 7.2\sqrt{2}\times10^{4}\ N/C \text{ at } 45^\circ \text{ to OA.}$$

Q. An electron falls through a distance of 1.5 cm in a uniform electric field of value 2×10^4 N/C. When the direction of electric field is reversed, a proton falls through the same distance. Compare the time of fall in each case.

Sol: (a) For electron



Acceleration $a_1 = \frac{F}{m_1} = \frac{q_1 E}{m_1}$

$$a_1 = \frac{1.6\times10^{-19}\times2\times10^4}{9\times10^{-31}}$$

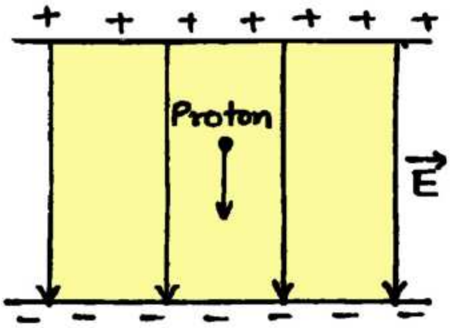
$$= 3.55\times10^{15}\ m/s^2$$

From $s = ut_1 + \frac{1}{2}a_1 t_1^2$

$s = 0 + \frac{1}{2}a_1 t_1^2$ or $t_1 = \sqrt{\frac{2s}{a_1}}$

$$t_1 = \sqrt{\frac{2\times1.5\times10^{-2}}{3.55\times10^{15}}} = 2.9\times10^{-9}\ sec.$$

(b) For Proton



Acceleration $a_2 = \frac{F}{m_2} = \frac{q_2 E}{m_2}$

$$a_2 = \frac{1.6\times10^{-19}\times2\times10^4}{1.67\times10^{-27}}$$

$$= 1.92\times10^{12}\ m/s^2$$

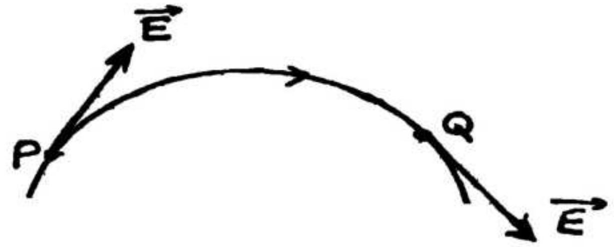
Similarly $t_2 = \sqrt{\frac{2s}{a_2}}$

$$= \sqrt{\frac{2\times1.5\times10^{-2}}{1.92\times10^{12}}} = 1.25\times10^{-7}\ sec.$$

so $\frac{t_1}{t_2} = \frac{2.9\times10^{-9}}{1.25\times10^{-7}} = 2.3\times10^{-2}$

Electric Field Lines

An electric field line is a curve drawn in such a way that the tangent to it at each point is in the direction of the net field at that point.



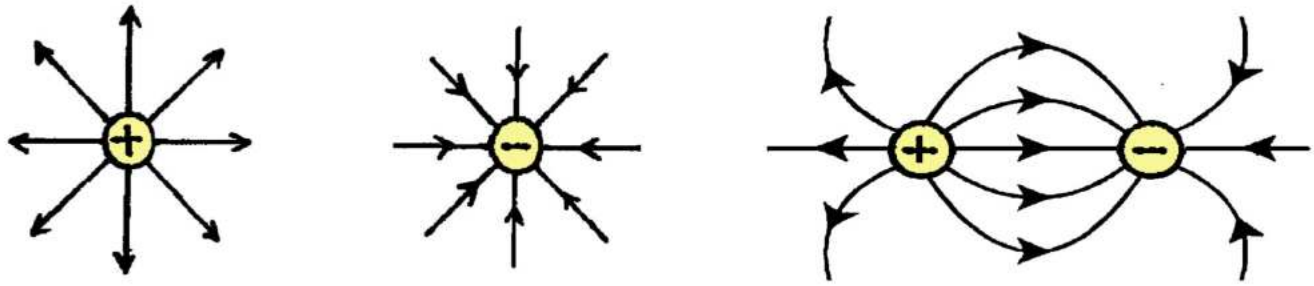
The magnitude of the field is represented by the density of field lines.

Hence Electric field intensity at a point is equal to number of field lines crossing normally a unit area around that point.

Properties of Electric Field Lines

(1) Electric field lines are continuous curves. They start from a positively charged body and end at a negatively charged body.

If there is a single charge then the field lines may start or end at infinity.

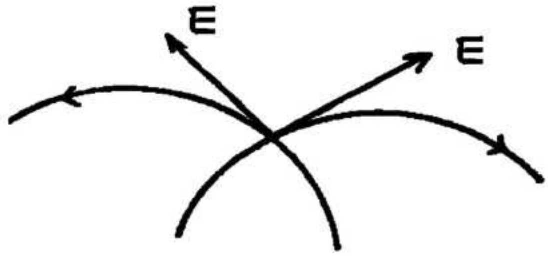


(2) No electric field lines exist inside the charged body. Thus electrostatic field lines do not form continuous closed loops.

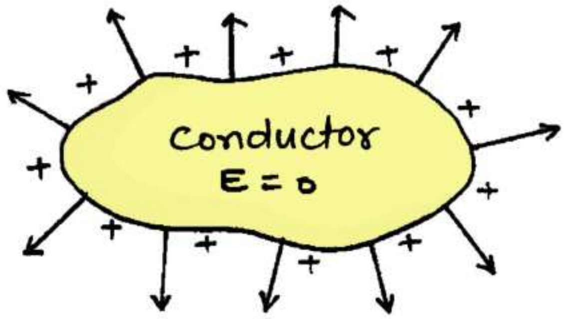
★ The magnetic field lines are continuous (or endless) closed loops as against the electric field lines.

(3) Tangent to the electric field line at any point gives the direction of electric field intensity at that point.

(4) No two electric field lines of force can intersect each other. This is because at the point of intersection P, we can draw two tangents PA and PB. This would mean two directions of electric field intensity at the same point, which is not possible.

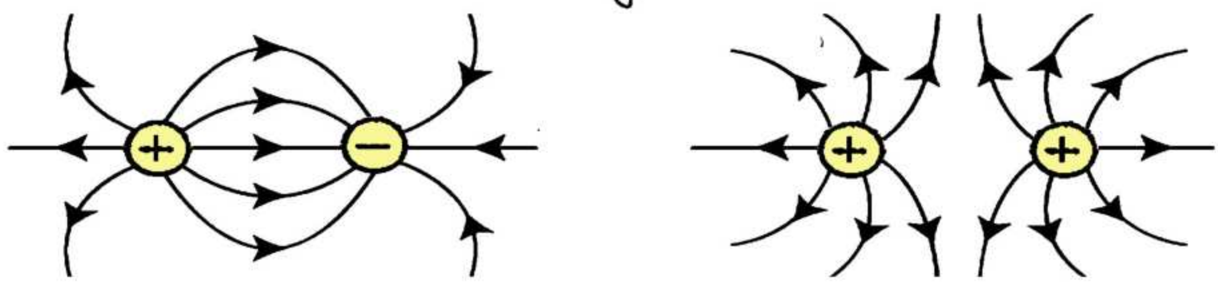


(5) The electric field lines are always normal to the surface of a conductor. So there is no component of electric field intensity parallel to the surface of the conductor.



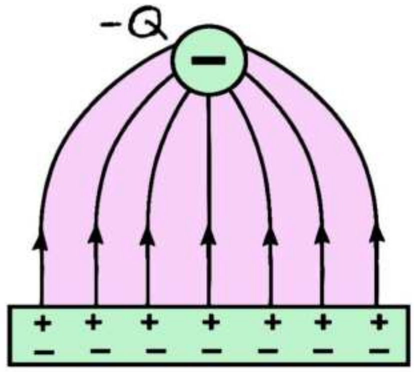
(6) The field lines around a system of two positive charges (q, q) gives a pictorial representation of their mutual repulsion.

While around the configuration of two equal and opposite charges (q, -q), a dipole, shows the mutual attraction between the charges.



Q. Draw the pattern of electric field lines when a point charge $-Q$ is kept near an uncharged conducting plate.
CBSE 2019

Sol. When $-Q$ charge is kept near an uncharged conducting plate, positive charge is induced due to electrostatic induction. The field lines will be perpendicular to the metal surface.



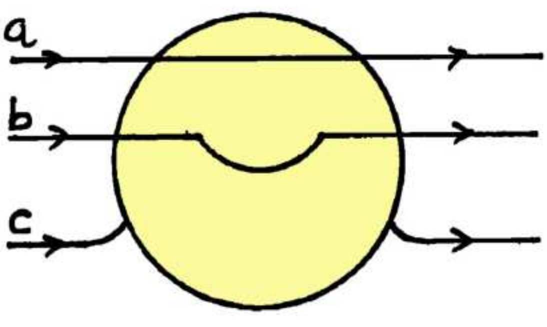
Q. Why do electrostatic field lines not form closed loops?
CBSE 2014,15

Sol. Electric field lines start from positive charge and terminate at negative charge. If there is a single positive charge the field lines start from the charge and terminate at infinity. So, the electric field lines do not form closed loops.

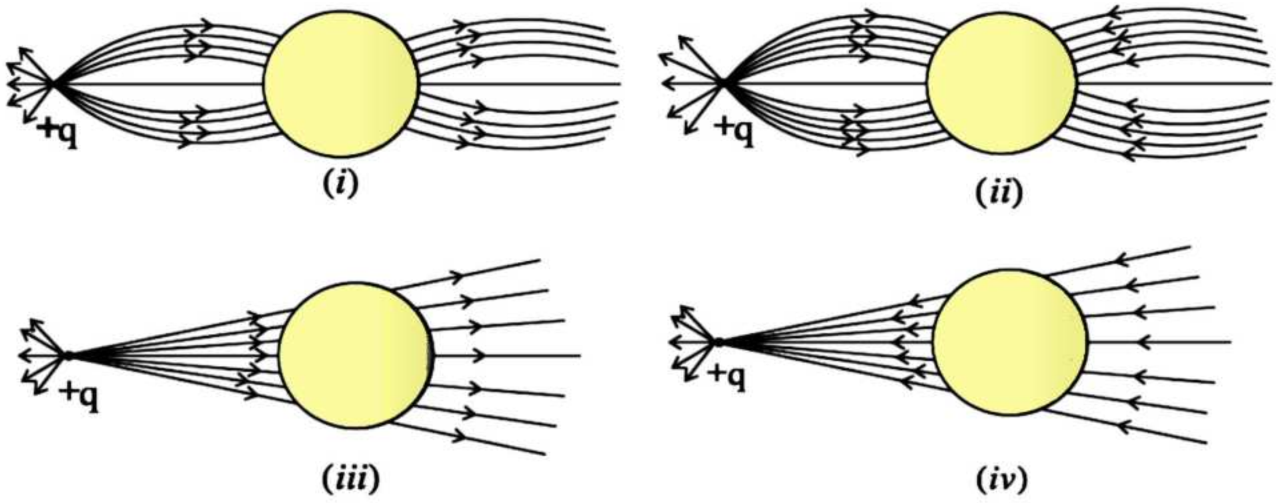
Q. A metal sphere is placed in a uniform electric field as shown in fig. Which path is followed by electric field lines and why?

Sol. Path c.

Electric field inside a metal sphere is zero, therefore no electric field lines exist inside the sphere. Also electric field lines are always perpendicular to the surface of conductor.



Q. (CBSE 2021) A point positive charge is brought near an isolated conducting sphere. The best representation of electric field is given by



Answers Fig (*i*)

Q. (CBSE 2014) An electric field line is a continuous curve. That is a field line cannot have sudden breaks. Why is it so?

Sol. An electric field line is the path of movement of a positive test charge ($q_0 \rightarrow 0$).

A moving charge experiences a continuous force in an electric field, so an electric field line is always a continuous curve.

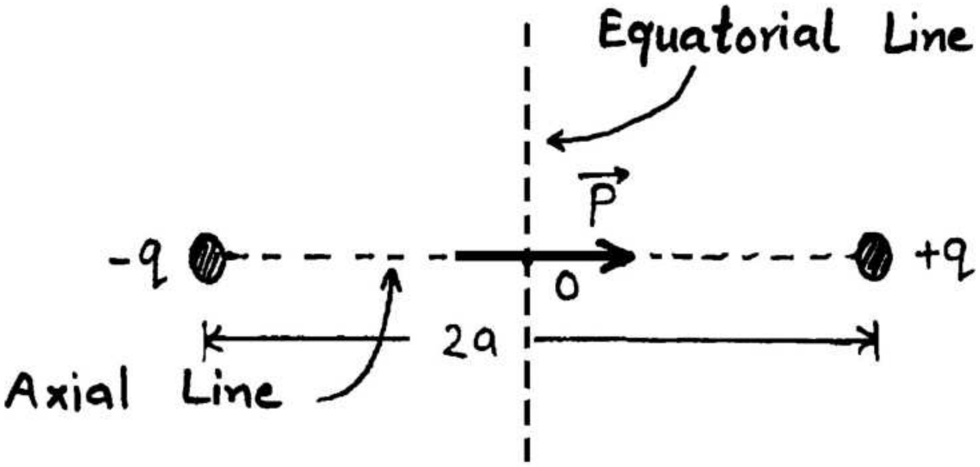
Q. (CBSE 2014, 15) Why do electrostatic field lines not form closed loops?

Sol. Electric field lines start from positive charge and terminate at negative charge. If there is a single positive charge the field lines start from the charge and terminate at infinity. So, the electric field lines do not form closed loops.

Electric Dipole

An electric dipole is a pair of equal and opposite point charges separated by a short distance.

Let q and $-q$ are two equal and opposite point charges separated by a small distance '$2a$'.



* The total charge of the electric dipole is zero, but this does not mean that the field of the electric dipole is zero.

Dipole Moment ($\vec{P}$)

[CBSE 2011, 2013, 2009, 2012, 2013]

Dipole moment is a measure of the strength of electric dipole. It is a vector quantity whose direction is from negative charge to positive charge.

Dipole moment is equal to the product of the magnitude of either charge and the distance between them.

$$\boxed{\vec{P} = q \times 2a \ \hat{P}}$$

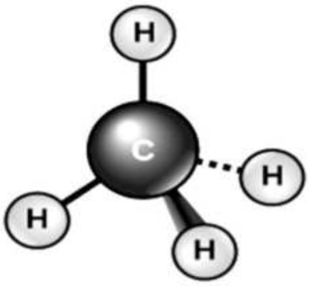
- Its SI unit is coulomb-metre.

Physical Significance of dipole

In most of the molecules the centres of positive and negative charges lie at the same place. Therefore, their dipole moment is zero.

CO_2 and CH_4 are of this type of molecules. However they develop a dipole moment when an electric field is applied. These types of molecules called non-polar molecules.

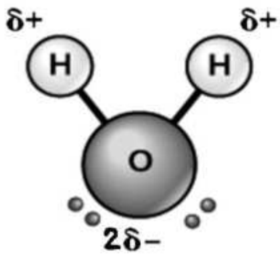
Linear

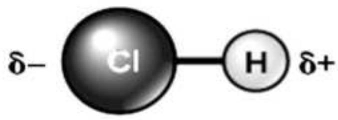


Tetrahedral

But in some molecules the centres of negative and positive charges do not coincide. Therefore they have a permanent electric dipole moment, even in the absense of of an electric field. Such molecules are called polar molecules.

Water molecules (H_2O) and HCl molecule are the examples of polar molecules.





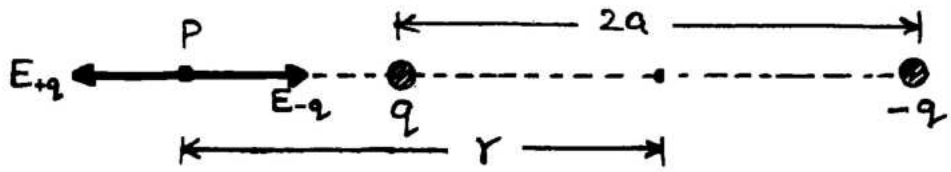
Dipole Field

It is the space around the dipole in which the electric effect of the dipole can be experienced.

The electric field at any point P is obtained by adding the electric fields E_{-q} due to charge $-q$ and E_{+q} due to the charge q by the parallelogram law of vectors.

(i) Field Intensity on Axial Line of Dipole

Let the point P be at distance r from the centre of the dipole on the side of the charge q, then



Here $E_{-q} = \dfrac{Kq}{(r+a)^2}$ and $E_{+q} = \dfrac{Kq}{(r-a)^2}$

So the total Electric field at point P is -

$$E = E_{+q} - E_{-q} \qquad (E_{+q} > E_{-q})$$

$$E = Kq\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]$$

$$E = Kq \cdot \frac{4ar}{(r^2-a^2)^2}$$

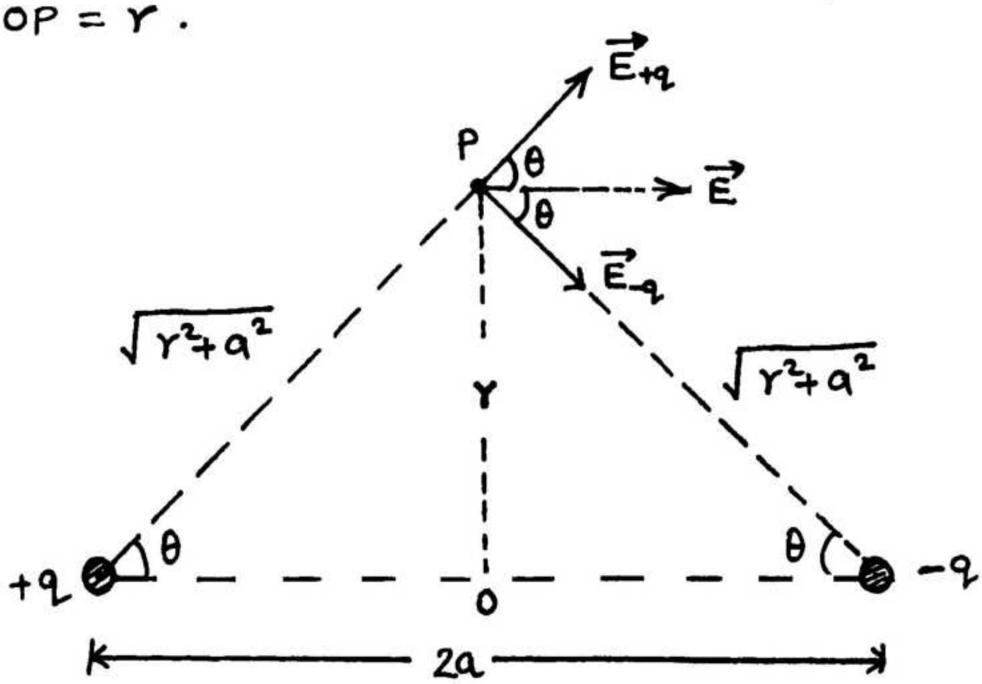
For $(r >> a)$ $\quad E = \dfrac{4Kqa}{r^3}$ $\qquad \boxed{\vec{E} = \dfrac{2K\vec{P}}{r^3}}$

The direction of $\vec{E}$ is along $\vec{P}$ i.e from $-q$ to $+q$.

* $\boxed{E \propto \dfrac{1}{r^3}}$

(ii) Field Intensity on Equatorial Line of Dipole

Let the point P is on the equatorial line, where OP = r.



So the magnitudes of the electric fields due to the two charges q and -q are given as-

$$E_{+q} = \frac{Kq}{(r^2+a^2)} \quad \text{and} \quad E_{-q} = \frac{Kq}{(r^2+a^2)}$$

Here the components of E_{+q} and E_{-q} normal to the dipole axis cancel each other and the components along the dipole axis add up.

So the total electric field is in opposite direction to $\hat{P}$.

$$E = E_{+q}\cos\theta + E_{-q}\cos\theta$$

or $$E = 2E_{+q}\cos\theta \Rightarrow E = \frac{K \cdot 2qa}{(r^2+a^2)^{3/2}}$$

For $(r >> a)$ $$E = \frac{K \cdot 2qa}{r^3}$$

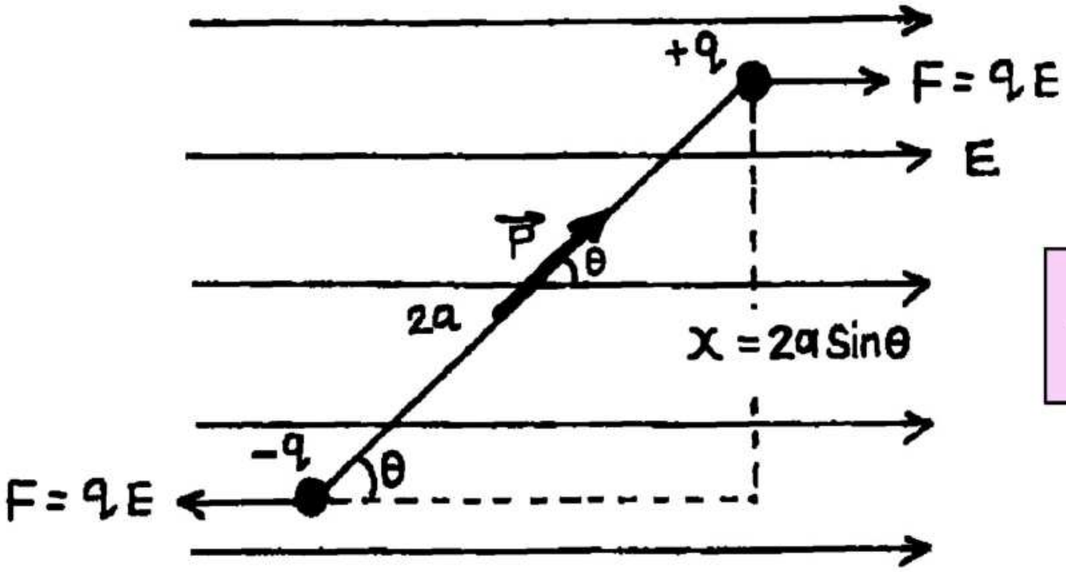
$$\boxed{\vec{E} = \frac{-K\vec{P}}{r^3}} \qquad \boxed{E \propto \frac{1}{r^3}}$$

* Hence the dipole field at large distances falls of not as $1/r^2$ but as $1/r^3$.

* Due to an electric dipole $\boxed{E_{axial} = 2\,E_{equatorial}}$

Dipole in a uniform Electric Field

Let a dipole be held in a uniform external electric field $\vec{E}$ at an angle θ with the direction of $\vec{E}$.



$$\sin\theta = \frac{x}{2a}$$

Net force on the dipole = zero

The forces on charge +q and -q resulting in a torque on the dipole which rotates the dipole.

Thus the torque tend to align the dipole axis along the direction of field $\vec{E}$.

The magnitude of torque is equals the magnitude of each force (qE) multiplied by the perpendicular distance between the two antiparallel forces.

$$\text{Torque} \quad \tau = F \times X$$

$$\tau = qE \times 2a\sin\theta$$

$$\tau = pE\sin\theta$$

or $$\boxed{\vec{\tau} = \vec{p} \times \vec{E}}$$

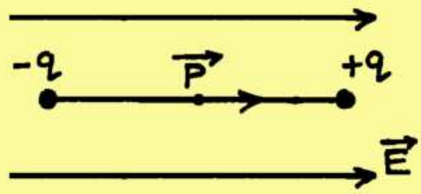
The direction of $\vec{\tau}$ is given by right handed screw rule and is perpendicular to $\vec{p}$ and $\vec{E}$.

CBSE 2017 **SPECIAL CASES -**

★ When $\vec{p}$ is along $\vec{E}$, $\theta = 0°$

$\tau = pE\sin 0° = 0$

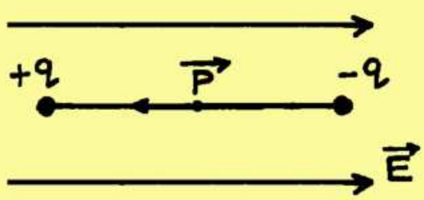
The dipole is in stable equilibrium.



★ When $\vec{p}$ is along $\vec{E}$, $\theta = 180°$

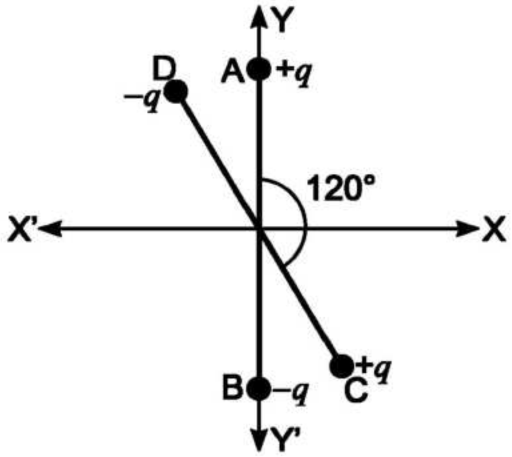
$\tau = pE\sin 180° = 0$

The dipole is in unstable equilibrium.



Q. (CBSE 2020,11) Two small identical electric dipoles AB and CD each of dipole moment 'p' are Kept at an angle 120°. as shown in fig. what is the resultant dipole moment of the system.

If this system is subjected to electric field $\vec{E}$ directed along +X direction, what will be the magnitude and direction of the torque acting on it.



Sol. Resultant dipole moment

$$P_r = \sqrt{P_1^2 + P_2^2 + 2P_1P_2\cos\theta}$$

$$P_r = \sqrt{p^2 + p^2 + 2p^2\cos 120^\circ}$$

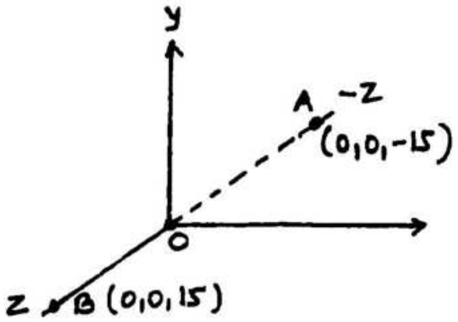
$$P_r = \sqrt{2p^2 + 2p^2 \times \left(-\tfrac{1}{2}\right)} = p$$

Resultant dipole moment makes an angle 60° with the Y-axis or 30° with the x-axis.

$$\text{Torque } \vec{\tau} = \vec{p} \times \vec{E}$$

$$\tau = pE\sin 30^\circ = \frac{1}{2}pE$$

Q. A system has two charges $q_A = 2.5 \times 10^{-7}$ C and $q_B = -2.5 \times 10^{-7}$ C Located at points A: (0, 0, −15 cm) and B: (0, 0, 15 cm) respectively. What are the total charge and electric dipole moment of the system?



Sol. Total charge of the system

$$q = q_A + q_B$$

$$q = 2.5 \times 10^{-7}\,C - 2.5 \times 10^{-7}\,C$$

$$q = 0$$

Electric dipole moment of the system is given by-

$$|\vec{P}| = q \times 2a = 2.5 \times 10^{-7} \times 0.30$$

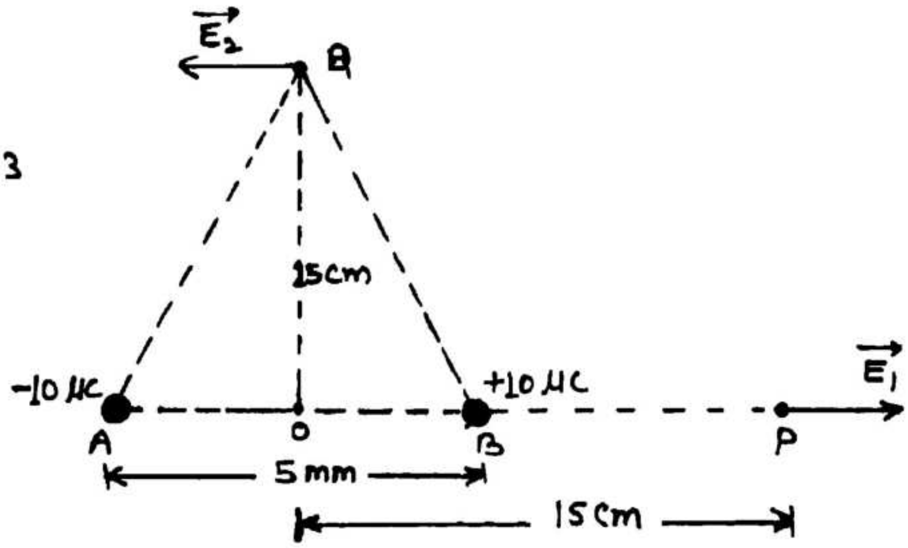
so $|\vec{P}| = 7.5 \times 10^{-8}$ C.m (along Positive Z-axis).

Q. Two charges $\pm 10\ \mu C$ are placed 5 mm apart. Determine the electric field at (a) a point P on the axis of dipole 15 cm. away from its centre on the side of the positive charge. (b) a point Q, 15 cm away from O on a line passing through O and normal to the axis of the dipole as shown in figure.

Sol. $|\vec{P}| = q \times 2a$

$|\vec{P}| = 10 \times 10^{-6} \times 5 \times 10^{-3}$

$= 5 \times 10^{-8}$ C.m.



(a) As point P lies on axial line of dipole so -

$$E_1 = \frac{2KP}{r^3} = \frac{2 \times 9 \times 10^9 \times 5 \times 10^{-8}}{(0.15)^3}$$

so $E_1 = 2.67 \times 10^5$ N/C (along the direction of $\vec{P}$)

(b) As point Q lies on equatorial line of dipole so –

$$E_2 = \frac{KP}{r^3} = \frac{9 \times 10^9 \times 5 \times 10^{-8}}{(0.15)^3}$$

$E_2 = 1.33 \times 10^5$ N/C (opposite to the direction of $\vec{P}$)

Q. In a certain region of space, electric field is along the z- direction. The magnitude of electric field is not Constant and increases uniformly along positive z-axis, at the rate of 10^5 N/c Per metre. Calculate the Force and torque experienced by a system having a total dipole moment 10^{-7} C.m. in the negative z direction?

Sol. Dipole moment of the system $|\vec{P}| = q \times dL$

$$|\vec{P}| = q \times dL = -10^{-7} \text{ C.m.}$$

Rate of increase of electric Field per unit length,

$$\frac{dE}{dL} = 10^5 \text{ N/c per meter}$$

Force experienced by the system is given by

$$F = qE$$

or $$F = q \times dL \times \frac{dE}{dL}$$

$$F = -10^{-7} \times 10^{-5} = -10^{-2} \text{ N} \quad (\text{along } -z \text{ axis})$$

Here the angle between electric field and dipole moment is 180°.

so Torque $\tau = PE\sin\theta$

$$\tau = PE\sin 180^\circ$$
$$= 0$$

Continuous charge distribution

In this distribution the charges are equally distributed in a body or object.

There are three types of continuous charge distribution -

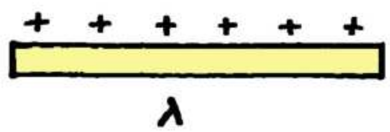
(a) Linear Charge Distribution -

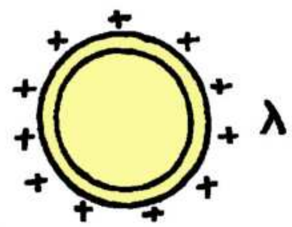
It is represented by linear charge density (λ).

$$\boxed{\text{Linear charge density } (\lambda) = \frac{\text{Charge}}{\text{Length}} = \frac{Q}{l}}$$

SI Unit $\rightarrow$ C/m.

Examples - Charged straight wire, Charged circular ring etc.





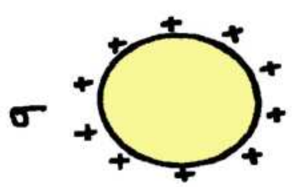
(b) Surface charge Distribution

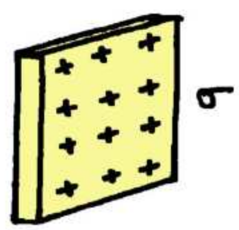
It is represented by surface charge density σ.

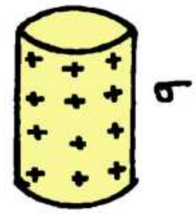
$$\boxed{\text{surface charge density } (\sigma) = \frac{\text{Charge}}{\text{Area}} = \frac{Q}{S}}$$

SI Unit $\rightarrow$ C/m^2.

Example - charged Plane sheet, Charge Conducting sphere, cylinder etc.





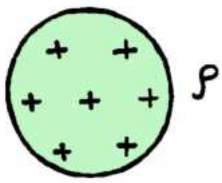


(c) Volume Charge Distribution

$$\text{Volume charge density } (\rho) = \frac{\text{Charge}}{\text{Volume}} = \frac{Q}{V}$$

SI Unit → C/m^3.

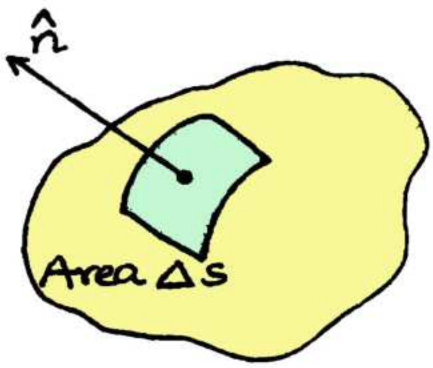
Examples – charged insulating sphare etc.



Area Vector

By convention, the vector associated with every area element of a closed surface is taken to be in the direction of the outward normal.

$$\boxed{\vec{S} = (s)\hat{n}}$$



Here Δs is magnitude of the area element and $\hat{n}$ is a unit vector in the direction of outward normal at that point.

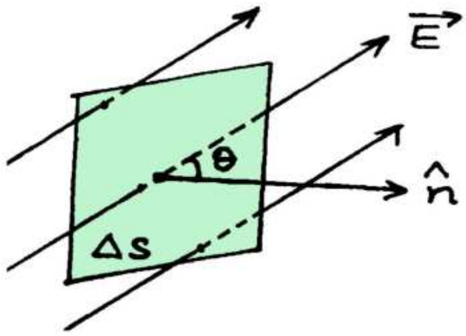
Electric Flux (ϕ)

Electric flux over an area in an electric field represents the total number of electric field lines crossing this area normally.

The electric flux $\Delta\phi$ through an area element $\Delta\vec{s}$ in an electric field $\vec{E}$ is defined as -

$$\boxed{\Delta\phi = \vec{E}\cdot\Delta\vec{s} = E(\Delta s)\cos\theta}$$

where θ is the angle between $\vec{E}$ and $\Delta\vec{s}$

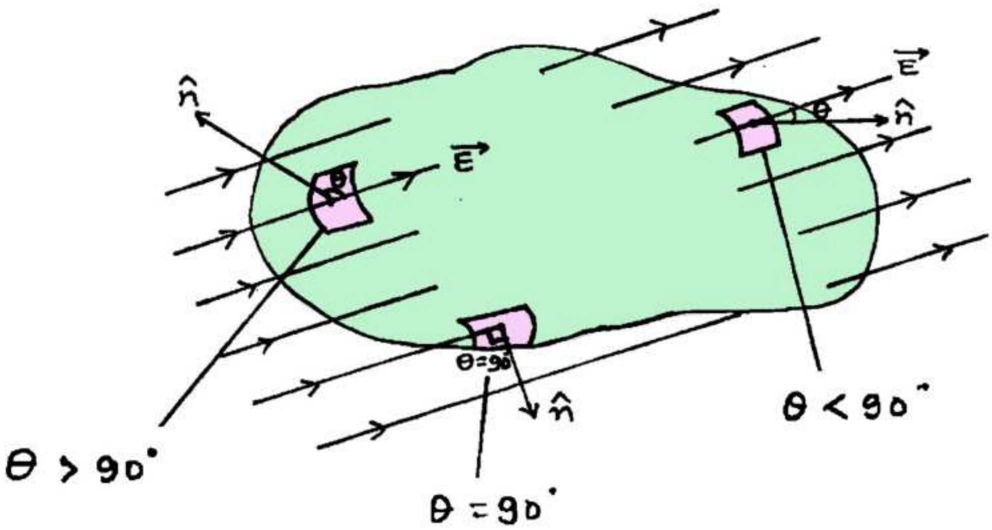


For a non uniform surface total electric flux is -

$$\boxed{\phi = \oint \vec{E}\cdot\vec{ds} = \oint E\,ds\cos\theta}$$

★ Electric flux is a scalar quantity.

★ For a closed surface the flux comes outside from the surface is taken **positive** and the flux going inside to the surface is taken **negative**.



Gauss's Law

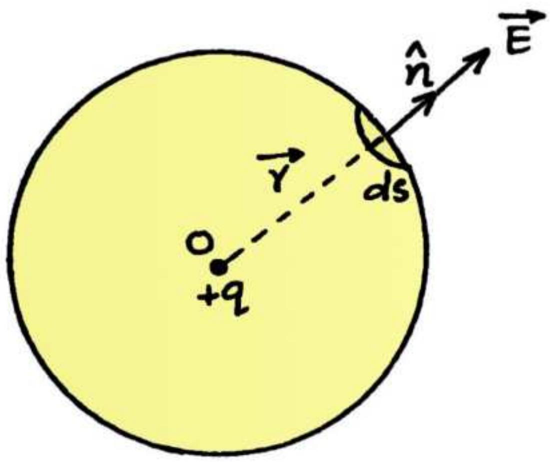
The total electric flux over the closed surface in vacuum is $1/\epsilon_0$ times the total charge Q contained inside surface.

$$\boxed{\phi = \oint_s \vec{E} \cdot \vec{ds} = \frac{Q}{\epsilon_0}}$$

Proof of Gauss's Law -

Suppose an isolated positive point charge q is situated at the centre O of a sphere of radius r.

The electric field intensity at any point P on the surface of the sphere is



$$\boxed{\vec{E} = \frac{Kq}{r^2}\hat{r}}$$

where $\hat{r}$ is unit vector directed from O to P.

Total electric flux over the entire surface of sphere

$$\phi = \oint_s \vec{E} \cdot \vec{ds} = \oint_s \frac{Kq}{r^2} \times ds \cos 0^\circ$$

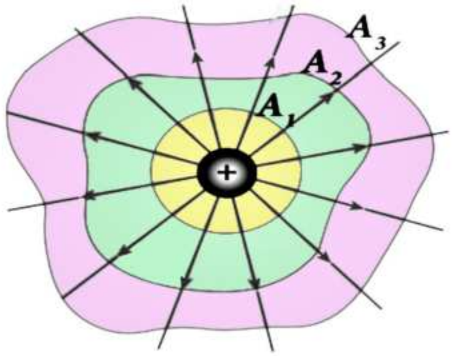
$$= \frac{Kq}{r^2} \oint_s ds$$

$$\phi = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} \times 4\pi r^2 = \frac{q}{\epsilon_0} \qquad \left(\because \oint_s ds = 4\pi r^2\right)$$

Hence $$\boxed{\phi = \oint_s \vec{E} \cdot \vec{ds} = \frac{q}{\epsilon_0}}$$

Important Points about Gauss's Law

1.| Gauss's Law is true for any closed surface, no matter what its shape or size.



The net electric flux is the same through all surfaces.

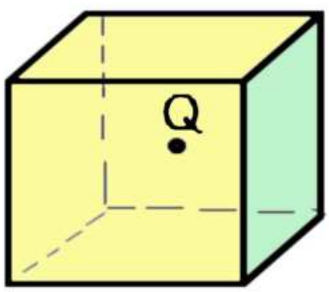
2.| The term q in Gauss's Law includes the sum of all charges enclosed by the surface. The charges may be located anywhere inside the surface.

3.| The Gaussian surface should not pass through any discrete charge, because electric field due to a system of discrete charges is not well defined, at the location of any charge.

However the Gaussian surface can pass through a continuous charge distribution.

4.| If the closed Gaussian surface have n symmetrical faces then the electric flux passing through one face will be $\frac{1}{n}$ times the total flux passing through the surface.

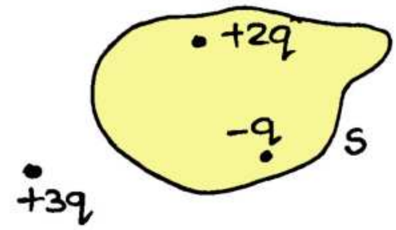
Example-



flux passing through from one face

$$\phi = \frac{1}{6}\frac{Q}{\epsilon_0}$$

Q. (CBSE 2010, 16) In the given fig. find the electric flux due to the charges +2q, −q and +3q, through the surface 's'.



Sol. Electric Flux = $\frac{1}{\epsilon_0} \times$ (Total charge within the surface)

$$\phi = \frac{1}{\epsilon_0} \times (2q - q) = \frac{q}{\epsilon_0}$$

Q. (CBSE 2015) What is the electric flux through a cube of side 1cm which encloses an electric dipole?

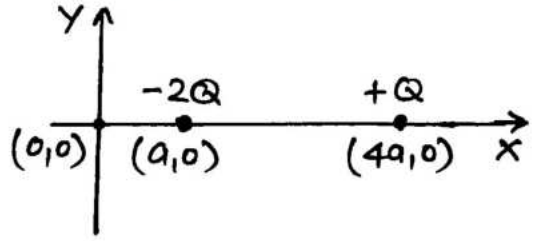
Sol. Net electric flux is zero.

Reason:

(i) Independent to the shape and size.
(ii) Net charge of the dipole is zero.

Q. (CBSE 2013) Two charges of magnitudes −2Q and +Q are located at points (a, 0) and (4a, 0) respectively. What is the electric flux due to these charges through a sphere of radius '3a' with its centre at the origin?

Sol.

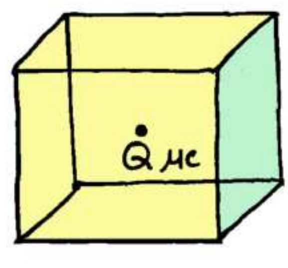


Total Electric flux $\phi = \frac{q_{in}}{\epsilon_0}$

$$\phi = \frac{-2Q}{\epsilon_0}$$

Q. (CBSE 2012, 17) A charge Q μC is placed at the centre of a cube. What would be the flux through one face?

Sol.



Electric flux through the whole cube

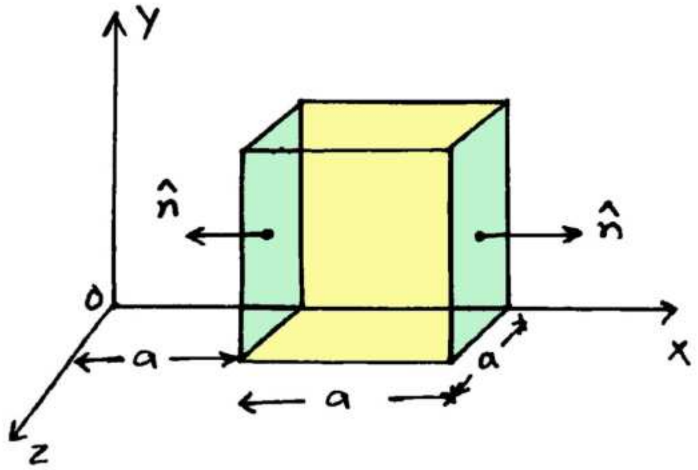
$$\phi = \frac{Q}{\epsilon_0}$$

Electric flux through one face

$$\phi' = \frac{1}{6}\frac{Q}{\epsilon_0}$$

Q. The electric field components in figure are $E_x = \alpha x^{1/2}$; $E_y = E_z = 0$ in which $\alpha = 800\ N/Cm^{1/2}$. Consider the cube as shown in fig and calculate -
(a) Total flux through the cube
(b) Charge within the cube. Assume that $a = 0.1\ m$.

Sol. Here $E_x = \alpha x^{1/2}$
$E_y = E_z = 0$,
$\alpha = 800\ N/C\text{-}m^{1/2}$, $a = 0.1\ m$



(a) The electric field has only x component, so
$\phi = \vec{E}\cdot\vec{ds} = 0$ for each of four faces of cube perpendicular to Y-Z plane.

At the left face $x = a$ so $E_L = \alpha a^{1/2}$

so $\phi_L = \vec{E_L}\cdot\vec{\Delta S} = \alpha a^{1/2} \times a^2 \cos 180^\circ = -\alpha a^{5/2}$

At the Right Face $x = 2a$ so $E_R = \alpha(2a)^{1/2}$

so $\phi_R = \vec{E_R}\cdot\vec{\Delta S} = \alpha(2a)^{1/2} \times a^2 \cos 0^\circ$

$\phi_R = \alpha a^{5/2}\sqrt{2}$

So Net flux through the cube $\phi = \phi_R + \phi_L$

$\phi = \alpha a^{5/2}\sqrt{2} - \alpha a^{5/2} = \alpha a^{5/2}(\sqrt{2}-1)$

$\phi = 800\,(0.1)^{5/2}(\sqrt{2}-1) = 1.05\ Nm^2/C$.

(b) By Gauss's theorem

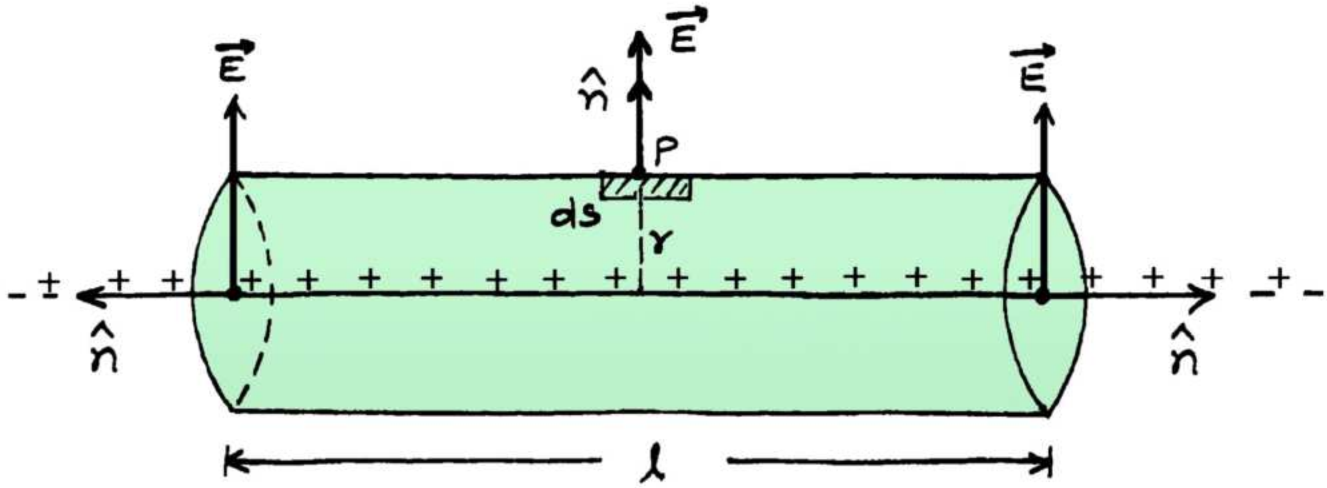
$q = \epsilon_0 \phi$

so $q = 8.85\times10^{-12} \times 1.05 = 9.27\times10^{-12}\ C$.

(1) Field due to an infinitely Long straight Uniformly charged wire -

Consider an infinitely long thin wire with uniform linear charge density λ.

The electric field at every point in the plane cutting the wire normally is radial and its magnitude depends only on the radial distance r.



Now consider a circular closed cylinder of radius r and length l, with the infinitely long line of charge as its axis.

So the total electric flux over the surface -

$$\phi = \oint_S \vec{E}\cdot\vec{ds} = \int_1 E\,ds\cos 0^\circ + \int_2 E\,ds\cos 90^\circ + \int_3 E\,ds\cos 90^\circ$$

$$\phi = E\int_3 ds \quad \text{or} \quad \phi = E \times 2\pi r l$$

Charge inside the cylinder $(q) = \lambda l$

Now according to Gauss's theorem $\phi = \dfrac{q}{\epsilon_0}$

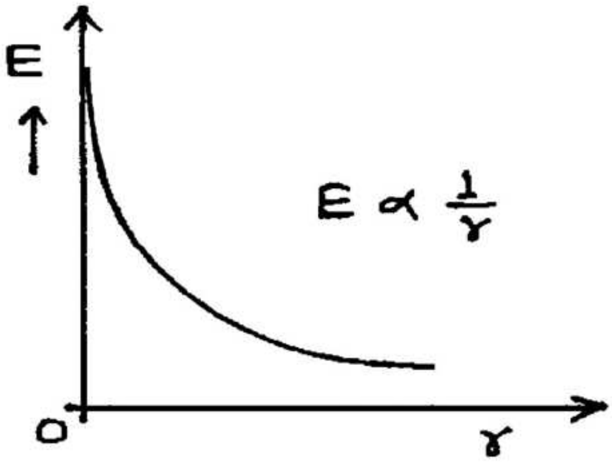
so $$E \times 2\pi r l = \frac{\lambda l}{\epsilon_0}$$

or $$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

or $$\boxed{\vec{E} = \frac{\lambda}{2\pi\epsilon_0 r}\hat{n}}$$ Clearly $E \propto \dfrac{1}{r}$

★ If $\lambda > 0$, the direction of electric field at every point is radially outwords.

★ If $\lambda < 0$, the direction of electric field at every point is radially inwards.



Q. A plastic rod of length 2.2 m and radius 3.6 mm carries a negative charge of 3.8×10^{-7} C which is spread uniformly over its surface. What is the electric field near the mid point of the rod, on its surface?

Sol: Given that – $l = 2.2$ m.

$r = 3.6$ mm $= 3.6 \times 10^{-3}$ m, $q = -3.8 \times 10^{-7}$ C.

Linear charge density $\lambda = \frac{q}{l}$

$$\lambda = \frac{-3.8 \times 10^{-7}}{2.2} = -1.73 \times 10^{-7} \text{ C/m}$$

As $$E = \frac{\lambda}{2\pi\epsilon_0 r} = \frac{2\lambda}{4\pi\epsilon_0 r}$$

So $$E = \frac{2 \times 9 \times 10^9 \times (-1.73 \times 10^{-7})}{3.6 \times 10^{-3}}$$

or $$E = -8.6 \times 10^5 \text{ N/C}$$

Q. An infinite line charge produces a field of 9×10^4 N/C at a distance of 4 cm. Calculate the linear charge density.

Sol. Here $E = 9\times10^4$ N/C , $r = 4\text{cm} = 4\times10^{-2}$ m

As $E = \frac{2K\lambda}{r}$

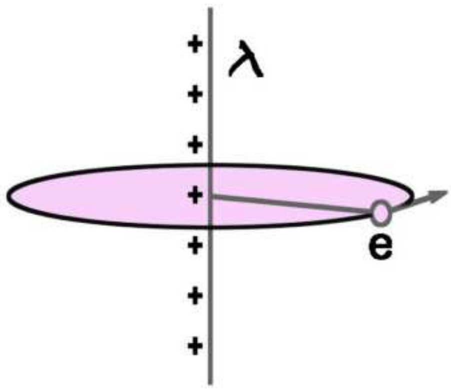
$$\lambda = \frac{E\times r}{2K} \Rightarrow \lambda = \frac{9\times10^4 \times 4\times10^{-2}}{2\times9\times10^9}$$

$$\lambda = 2\times10^{-7}\ \text{C/m}$$

Q. (CBSE 2013, 19) An infinitely long positively charged straight wire has a linear charge density λ C/m. An electron is revolving around the wire as its centre with a constant velocity in a circular plane perpendicular to the wire. Deduce the expression for its kinetic energy.

Sol. Infinitely long charged wire produces a radial Electric field

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$



The revolving electron experiences an electrostatic force and provides necessary centripetal force

$$eE = \frac{mv^2}{r}$$

$$e\times\frac{\lambda}{2\pi\epsilon_0 r} = \frac{mv^2}{r} \Rightarrow mv^2 = \frac{e\lambda}{2\pi\epsilon_0}$$

$$\text{Kinetic Energy} = \frac{1}{2}mv^2 = \frac{e\lambda}{4\pi\epsilon_0}$$

(2) Electric Field Intensity due to a Uniformly Charged spherical shell

(a) Electric Field Outside the shell -

To calculate electric field intensity at any point P, imagine a sphere (Gaussian surface) with centre O and radius r.

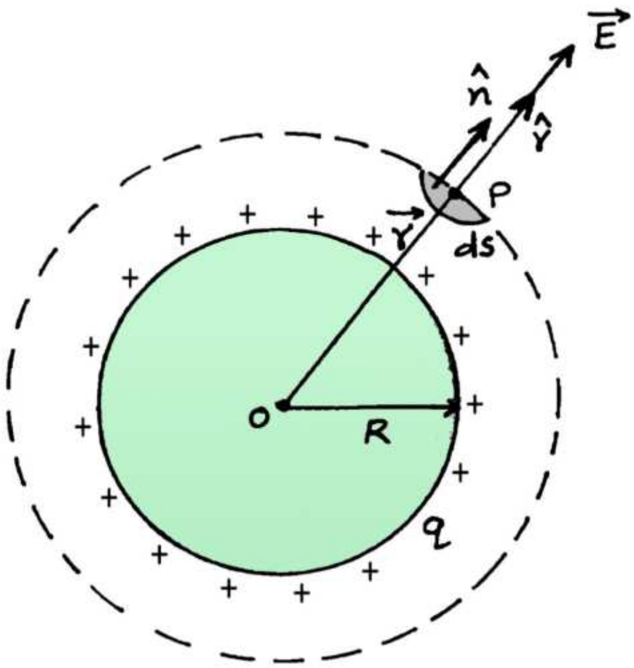
According to Gauss's Law

$$\phi = \oint_s \vec{E}\cdot\vec{ds} = \frac{q}{\epsilon_0}$$

or $$\phi = \oint_s E\, ds \cos 0^\circ = \frac{q}{\epsilon_0}$$

$$E \oint_s ds = \frac{q}{\epsilon_0}$$

$$\boxed{E = \frac{q}{4\pi\epsilon_0 r^2}}$$



* If $q > 0$ then $\vec{E}$ is radially outwards
* If $q < 0$ then $\vec{E}$ is radially inwards

(b) At a point on the surface of the shell -

At the surface of the spharical shell

$r = R$ so $$\boxed{E = \frac{q}{4\pi\epsilon_0 R^2}}$$

If σ is surface charge density on the shell then-

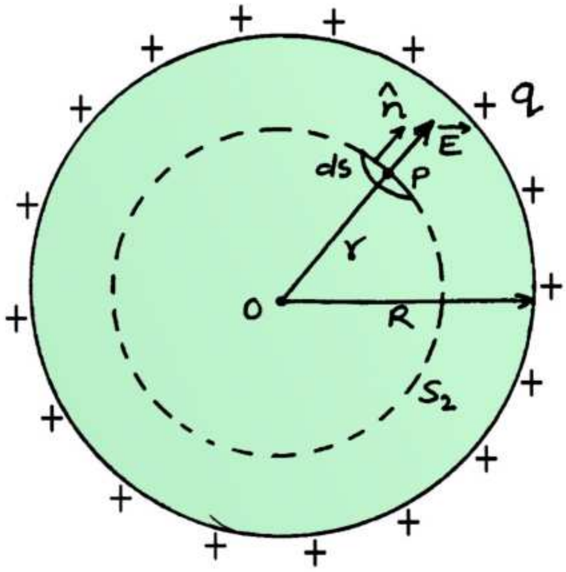
$$\boxed{E = \frac{\sigma}{\epsilon_0} = \text{Constant}} \qquad \left(\sigma = \frac{q}{4\pi R^2}\right)$$

(c) Electric Field inside the shell -

To find field at point P inside the shell consider a Gaussian surface of a sphare S_2 passing through P and with centre at O. Here $r < R$.

As the charge inside the spherical shell is zero the Gaussian surface encloses no charge.

By Gauss's theorem-

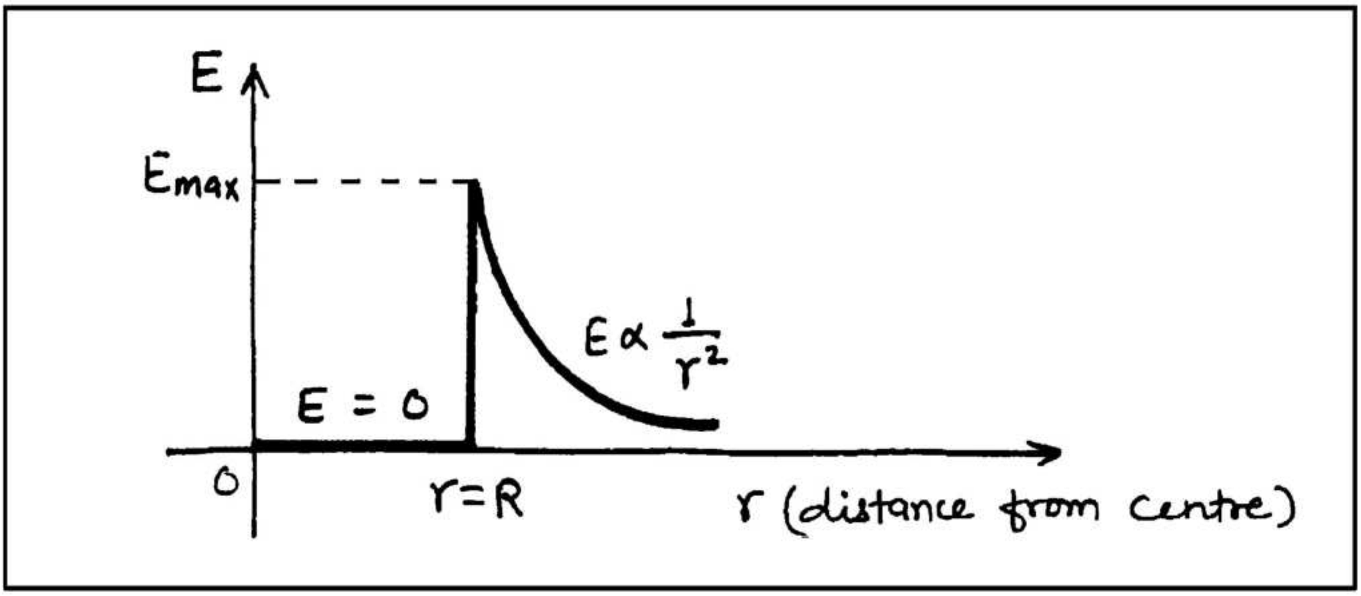


$$\phi = \oint_s \vec{E}\cdot\vec{ds} = \frac{q}{\epsilon_0}$$

$$\phi = E \times 4\pi r^2 = 0$$

so $\boxed{E = 0}$ for $r<R$

Hence the electric field inside a uniformly charged spherical shell is zero



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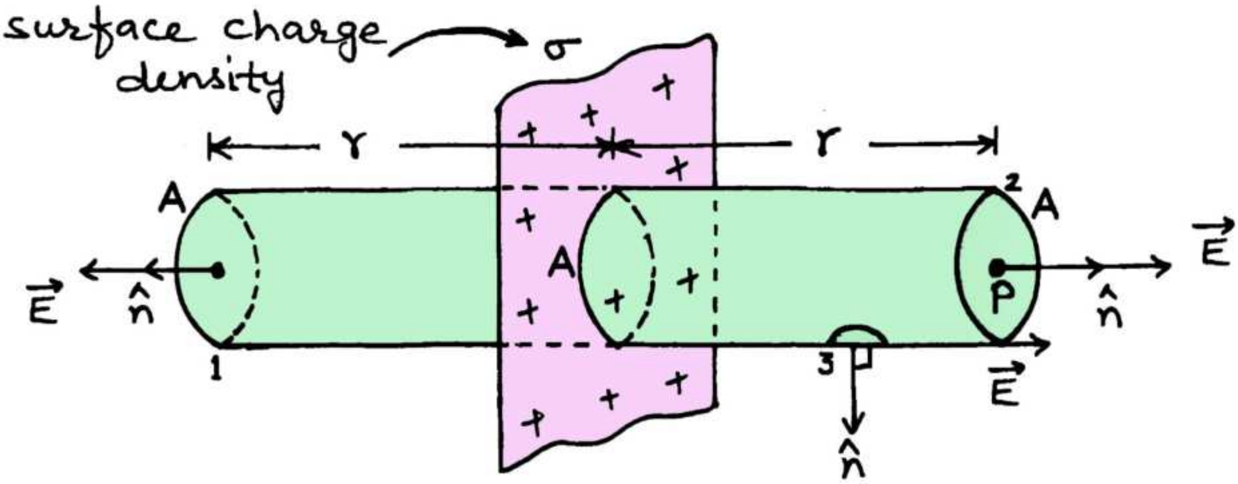
(3) Electric Field intensity due to a thin infinite plane sheet of charge

We have to find the electric field at point P at a perpendicular distance r from the sheet.

Now imagine a cylinderical Gaussian surface of cross-sectional area A around P and length 2r, passing through the sheet.



Total flux over the entire surface of cylinder is-

$$\phi = \int_1 \vec{E}\cdot\vec{ds} + \int_2 \vec{E}\cdot\vec{ds} + \int_3 \vec{E}\cdot\vec{ds}$$

$$\phi = \int_1 E\,ds\cos 0^\circ + \int_2 E\,ds\cos 0^\circ + \int_3 E\,ds\cos 90^\circ$$

$$\phi = 2EA$$

Total Charge enclosed by the cylinder = σA

According to Gauss's Law- $\phi = \frac{q}{\epsilon_0}$

$$2EA = \frac{\sigma A}{\epsilon_0} \qquad E = \frac{\sigma}{2\epsilon_0}$$

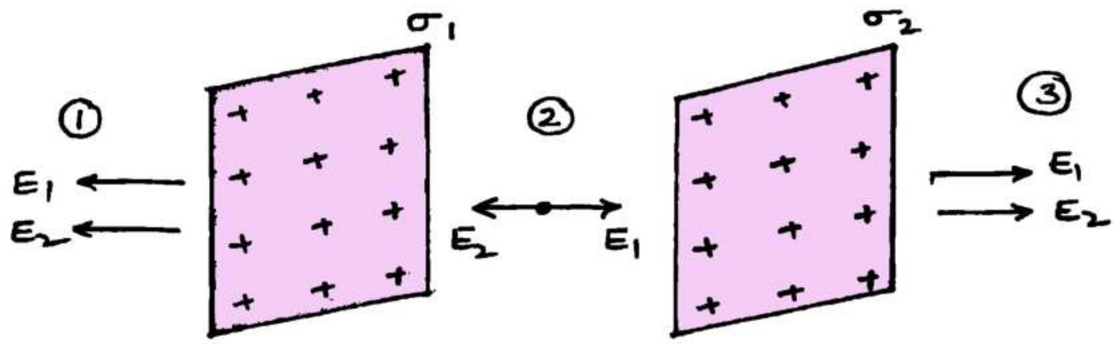
so $\boxed{\vec{E} = \frac{\sigma}{2\epsilon_0}\hat{n}}$ ★ E is independent of r.

★ If $\sigma > 0$ then electric field is normally outward

★ If $\sigma < 0$ then electric field is normally inward

(4) Electric Field intensity due to two thin infinite parallel sheets of charge -

Case 1:- Here $\sigma_1 > \sigma_2 > 0$



In region ① & ③

$$E = E_1 + E_2$$

$$E = \frac{\sigma_1}{2\epsilon_0} + \frac{\sigma_2}{2\epsilon_0}$$

$$\boxed{E = \frac{1}{2\epsilon_0}(\sigma_1 + \sigma_2)}$$

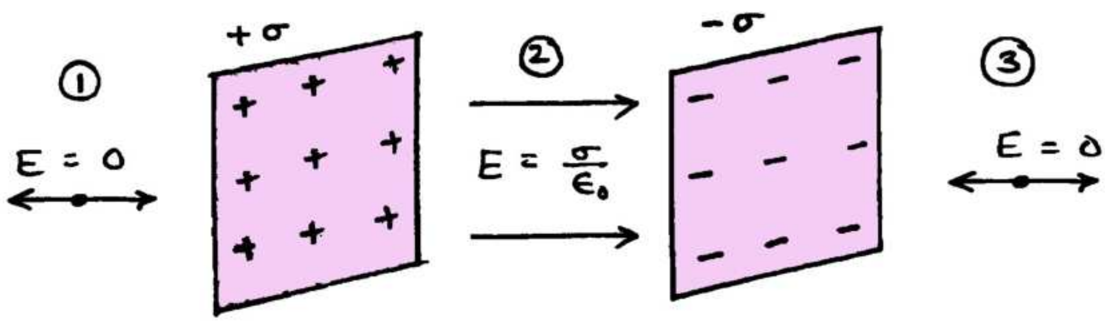
In region ②

$$E = E_1 - E_2$$

$$E = \frac{\sigma_1}{2\epsilon_0} - \frac{\sigma_2}{2\epsilon_0}$$

$$\boxed{E = \frac{1}{2\epsilon_0}(\sigma_1 - \sigma_2)}$$

Case 2:- Here $\sigma_1 = \sigma$ and $\sigma_2 = -\sigma$



In region ① & ③

$$E = E_1 - E_2$$

$$E = \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0}$$

$$\boxed{E = 0}$$

In region ②

$$E = E_1 + E_2$$

$$E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0}$$

$$\boxed{E = \frac{\sigma}{\epsilon_0}}$$

Q. A Large plane sheet of charge having surface charge density 5×10^{-6} C/m² lie in XY plane. Find the electric flux through a circular area of radius 0.1 m if the normal to the circular area makes an angle of 60° with the z-axis.

Sol. Here $\sigma = 5 \times 10^{-6}$ C/m², $r = 0.1$ m, $\theta = 60°$

$$\text{so} \quad \phi = \vec{E} \cdot \vec{ds} = E\,ds\,\cos\theta$$

$$\phi = \left(\frac{\sigma}{2\epsilon_0}\right) \times \pi r^2 \cos\theta$$

$$\phi = \frac{5 \times 10^{-6}}{2 \times 8.85 \times 10^{-12}} \times 3.14 \times (0.1)^2 \cos 60°$$

$$\phi = 4.44 \times 10^{3} \ \text{Nm}^2/\text{C} \quad \text{— Ans.}$$

Q. A charge of 17.7×10^{-4} C is distributed over a large sheet of area 200 m². Calculate the electric field at a distance of 20 cm from it.

Sol. Surface charge density $\sigma = \frac{Q}{A}$

$$\sigma = \frac{17.7 \times 10^{-4}}{200} = 8.85 \times 10^{-6} \ \text{C/m}^2$$

Electric field $E = \frac{\sigma}{2\epsilon_0}$

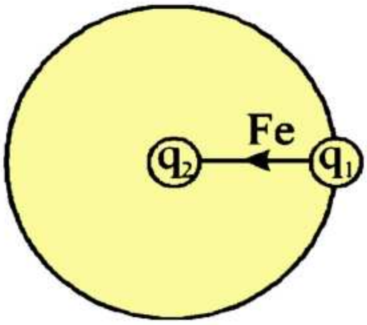
$$E = \frac{8.85 \times 10^{-6}}{2 \times 8.85 \times 10^{-12}} = 5 \times 10^{5} \ \text{N/C}$$

Q.1. A particle of charge q_1 and mass m is revolving around a fixed negative charge of magnitude q_2 in a circular path of radius r. Find the time period of revolution.

Solution :

The force of attraction between q_1 and q_2 provides necessary centripetal force.

Hence $F_e = F_c$



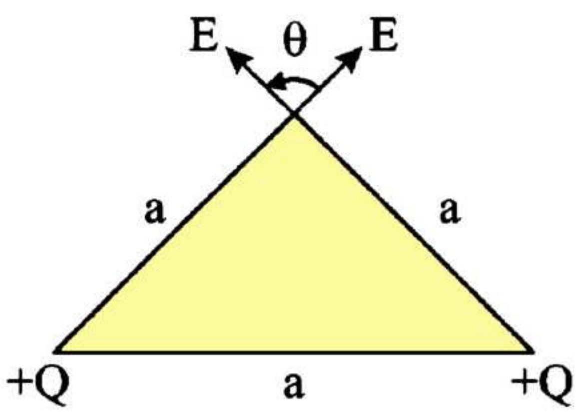
$$\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2} = m\omega^2 r$$

$$\Rightarrow \omega = \sqrt{\frac{q_1 q_2}{4\pi\epsilon_0 m r^3}} \quad \text{and Time period } T = \frac{2\pi}{\omega}$$

$$T = 4\pi r\sqrt{\frac{\pi\epsilon_0 m r}{q_1 q_2}}$$

Q.2. Two charges +Q each are placed at the two vertices of an equilateral triangle of side a. The intensity of electric field at the third vertex is

Solution:



$$E^1 = \sqrt{E^2 + E^2 + 2\times E\times E\times\cos\theta}$$

$$= \sqrt{2E^2 + 2E^2\cos\theta} \quad = \sqrt{2E^2(1+\cos\theta)}$$

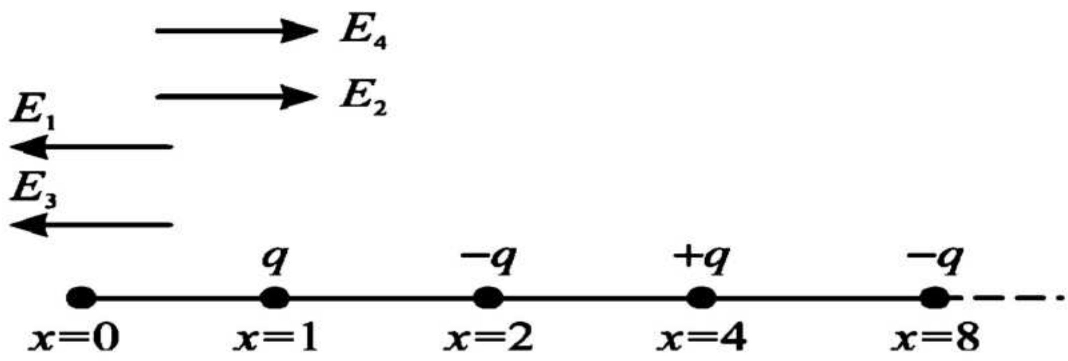
$$= 2E\cos\frac{\theta}{2}; \quad \text{Here } \theta = 60^0$$

$$\therefore \quad E = \sqrt{3}\frac{1}{4\pi\in_0}\frac{Q}{a^2}$$

Q.3. An infinite number of charges each 'q' are placed in the x-axis at distances of 1,2,4,8...meter from the origin. If the charges are alternately positive and negative find the intensity of electric field at origin.

Solution:

The electric field intensities due to positive charges and due to -ve charges the field intensity is towards the charges



The resultant intensity at the origin

$$E = E_1 - E_2 + E_3 - E_4 - - - - -$$

$$E = \frac{Q}{4\pi\in_0}\left(1 - \frac{1}{2^2} + \frac{1}{4^2} - \frac{1}{8^2} + \ldots\ldots\right)$$

Since the expression in the bracket is in GP

with a common ratio $= \frac{-1}{2^2} = \frac{-1}{4}$

$$E = \frac{Q}{4\pi\in_0}\frac{1}{\left[1 - \left(\frac{-1}{4}\right)\right]} = \frac{Q}{4\pi\in_0}\frac{4}{5}$$

$$E=\frac{4}{5}\frac{Q}{4\pi\in_0}$$

$$E=\frac{Q}{5\pi\in_0}$$

Q.4. *An α particle is located at a point where electric field strength is 3×10^4 N/C. Calculate (a) the force on the α-particle (b) its acceleration.*

Solution :

(a) $\vec{F}=q\vec{E}$

$$\vec{F}=\left(2\times1.6\times10^{-19}\right)C\times3\times10^4\,(N/C)$$

$$=6\times1.6\times10^{-15}=9.6\times10^{-15}\,N$$

(b) $$\vec{a}=\frac{\vec{F}}{m}=\frac{q\vec{E}}{m}=\frac{9.6\times10^{-15}\quad N}{4\times1.6\times10^{-27}\,kg}\,us$$

$$=1.5\times10^{12}\,m/s^2$$

Q.5. *A pendulum bob has mass 4 mg and carries a charge 2×10^{-9} coulomb. It hangs in equilibrium from a massless thread of length 50 cm whose other end is fixed to a vertical wall. A horizontal electric field of intensity 200 V/cm exists in space. Calculate*

(a) Angle made by the thread with the vertical

(b) Tension in the thread

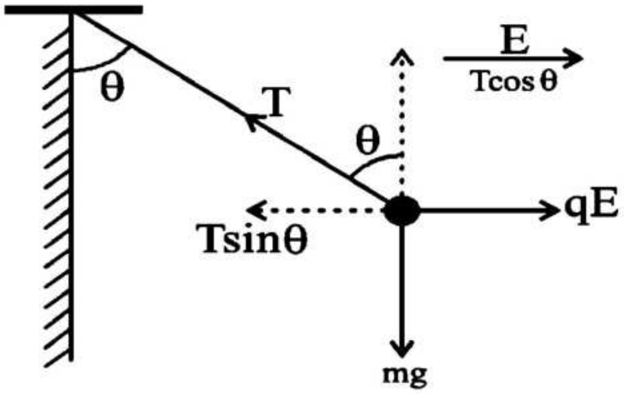
Solution :

(a) At equilibrium, $qE=T\sin\theta$

$mg=T\cos\theta$ Therefore,

$$\tan\theta=\frac{qE}{mg}=\frac{2\times10^{-9}\times2\times10^4}{4\times10^{-6}\times10}=1.0$$

$$\theta=45^\circ$$



(b) Tension T, electric force F_e and gravity force mg act on the bob, as shown $\vec{T} + q\vec{E} + m\vec{g} = 0$
Also T balances the resultant of qE and mg.

Therefore $T = \sqrt{(qE)^2 + (mg)^2}$

$$= \sqrt{2}\, qE \qquad \text{(as } qE = mg\text{)}$$

$$= \sqrt{2} \times 2 \times 10^{-9} \times 2 \times 10^{4}$$

$$= 5.64 \times 10^{-5}\ N$$

Q.6. A charge q = 1 μC is placed at point (1m, 2m, 4m). Find the electric field at point P(0m, –4m, 3m).

Solution : Here, $\vec{r}_q = \hat{i} + 2\hat{j} + 4\hat{k}$ and $\vec{r}_p = -4\hat{j} + 3\hat{k}$

$$\therefore \qquad \vec{r}_p - \vec{r}_q = -\hat{i} - 6\hat{j} - \hat{k}$$

or $\qquad |\vec{r}_p - \vec{r}_q| = \sqrt{(-1)^2 + (-6)^2 + (-1)^2} = \sqrt{38}$ m

Now, $\qquad \vec{E} = \dfrac{1}{4\pi \in_0} \dfrac{q}{|\vec{r}_p - \vec{r}_q|^3} \left(\vec{r}_p - \vec{r}_q\right)$

Substituting the values, we have

$$\vec{E} = \frac{(9 \times 10^9)(1.0 \times 10^{-6})}{(38)^{3/2}} (-\hat{i} - 6\hat{j} - \hat{k})$$

$$\vec{E} = (-38.42\hat{i} - 231.52\hat{j} - 38.42\hat{k})\ \frac{N}{C}$$

Q.7. Find out the torque on dipole in N-m given : Electric dipole moment $\vec{P}=10^{-7}(5\hat{i}+\hat{j}-2\hat{k})$ coulomb meter and electric field $\vec{E}=10^{7}(\hat{i}+\hat{j}+\hat{k})$ Vm^{-1} is -

Solution:

$$\vec{\tau}=\vec{P}\times\vec{E}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 5 & 1 & -2\\ 1 & 1 & 1\end{vmatrix}$$

$$=\hat{i}(1+2)+\hat{j}(-2-5)+\hat{k}(5-1)\ =3\hat{i}-7\hat{j}+4\hat{k}$$

$$|\vec{\tau}|=8.6\text{ N-m}$$

Q.8. What is the value of electric flux in SI unit in Y-Z plane of area 2m², if intensity of electric field is $\vec{E}=(5\hat{i}+2\hat{j})$ N/C.

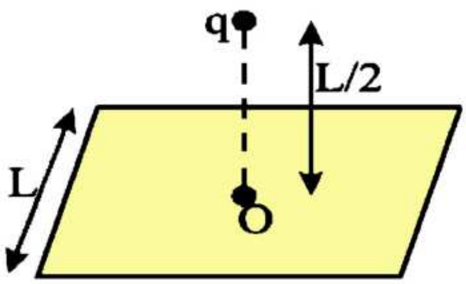
Solution:

$$\phi=\vec{E}.d\vec{A}\ =(5\hat{i}+2\hat{j}).2\hat{i}=10\ \frac{N}{C}m^2$$

Q.9. A point charge +q is located L/2 above the centre of a square having side L. Find the flux through this square.

Solution:

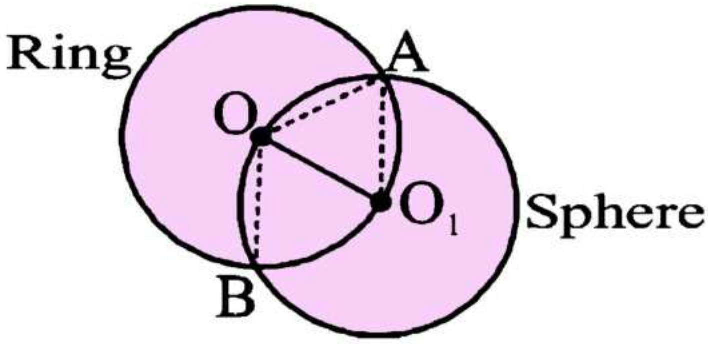
The charge q can be supposed to be situated at the centre of a cube having side L with outward flux ϕ. In this cube the square is one of its face having flux $\phi/6$.



$\therefore$ Flux through the square

$$=\frac{q}{6\varepsilon_0}$$

Q.10. A charge Q is distributed uniformly on a ring of radius r. A sphere of equal radius r is constructed. With its centre at the periphery of the ring (fig.) Find the flux of the electric field through the surface of the sphere.



Solution:

From the geometry of the fig. $OA = OO_1$ and $O_1A = O_1O$. Thus, OAO_1 is an equilateral triangle.

Hence $\angle AOO_1 = 60^0$ OR $\angle AOB = 120^0$

The arc AO_1B of the ring subtends an angle 120^0 at the centre O. Thus, one third of the ring is inside the sphere. The charge enclosed by the sphere $= \frac{Q}{3}$. Thus flux out of sphere $\frac{Q}{3\epsilon_0}$