Electrostatics

Electric Charge -

It is the property associated with matter due to which it produces and experiences electrical and magnetic effects.

The excess or dificiency of electrons in a body gives the concept of charge.

SI Unit - Coulomb (C) or Ampere x Sec. (A x s)

Dimensions - [AT]

Basic Properties of Charge -

1.| Charge is a scalar quantity.

2.| Charge is always associated with mass.

* when body gets +ve charge its mass decreases.
* when body gets -ve charge its mass increases.

3.| Charge is quantised. $\boxed{Q = \pm ne}$

where n = any integer value and $e = 1.6 \times 10^{-19}\,C$

4.| Charge is conserved. (But mass is not conserved)

5.| Similar charges repel and dissimilar charges attract each other.

6.| Accelerated charge radiates energy.

$V = 0$	$V = $ Const.	$V \neq$ Const.
Q ⊕	Q ⊕	Q ⊕
Produces only $\vec{E}$	Produces only $\vec{E}$ & $\vec{B}$	Produces $\vec{E}$ & $\vec{B}$ and radiates energy

7.| True test of electrification is repulsion, not attraction.

8.| Charge given to a conductor always resides on its surface. In insulator it remains where it is placed i.e. may reside inside the body also.

9.| Charge density of a irregular conducting body is not uniform. It is maximum where the radius of curvature is minimum and vice-versa. i.e. $\sigma \propto \frac{1}{R}$.
This is why charge leaks from sharp points.

Charge density $\boxed{\sigma \propto \frac{1}{R}}$

10.| The apparatus used to detect charge on a body is "Gold Leaf Electroscope".

Methods of Charging -

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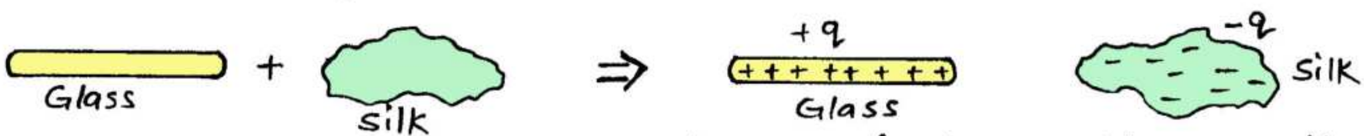
1.| Friction -

By rubbing two bodies together both positive and negative charges in equal amount appears due to transfer of electron from lower work function body to body of higher work function.

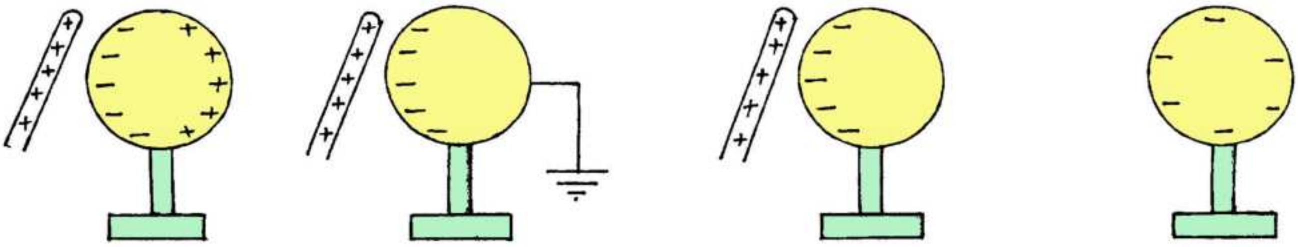


2.] Conduction -

The process of transfer of charge by contact of two bodies is known as conduction.
Positive charge flows from higher potential to lower potential, while negative charge flows from lower to higher potential.

3.] Induction -

If a charged body is brought near a neutral body, the charged body will attract opposite charge and repel similar charge present in the neutral body. As a result one side of the neutral body becomes positive and the other negative, this process is known induction.



(a) (b) (c) (d)

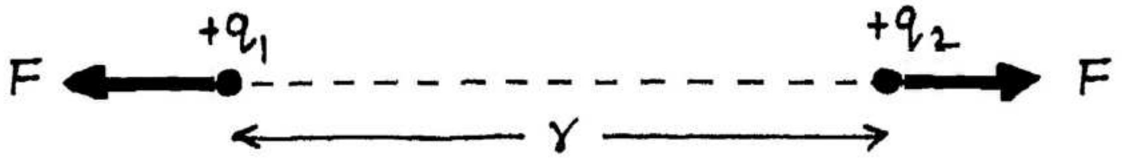
Induced charge can be lesser or equal to inducing charge (but never greater) and its maximum value is given by

$$\boxed{Q' = -Q\left[1 - \frac{1}{K}\right]}$$

$K \rightarrow$ dielectric Const.

Coulomb's Law -

The force of interaction between any two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them, and acts along the line joining the two charges.



According to coulomb's Law,

$$F \propto \frac{q_1 q_2}{r^2}$$

or $$\boxed{F = \frac{K q_1 q_2}{r^2}}$$ K - Electrostatic Constant

* Electrostatic constant K depends on the nature of medium separating the charges and on the system of units.

In SI system, $K = \frac{1}{4\pi\epsilon_0} = 9 \times 10^{9}\ Nm^2/C^2$

* ϵ_0 electrical permittivity of free space

$$\epsilon_0 = 8.85 \times 10^{-12}\ C^2/Nm^2$$

Vector form of coulomb's Law -

Force on q_1 due to q_2 = $\vec{F}_{12} = \frac{K q_1 q_2}{r_{21}^2}\hat{r}_{21}$

Force on q_2 due to q_1 = $\vec{F}_{21} = \frac{K q_1 q_2}{r_{12}^2}\hat{r}_{12}$

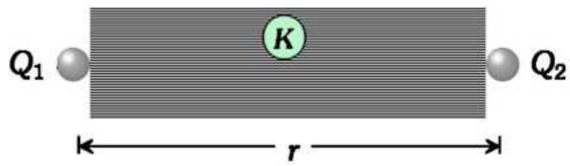
Here $\hat{r}_{12}$ and $\hat{r}_{21}$ are unit vectors from q_1 to q_2 and q_2 to q_1.

Effect of medium on Coulomb's Force -

(a) When a dielectric medium is completely filled in between charges, rearrangement of charges inside the dielectric medium take place and the force between the charges decreases by a factor of K (dielectric Const.)

$$F_{medium} = \frac{F_{air}}{K}$$

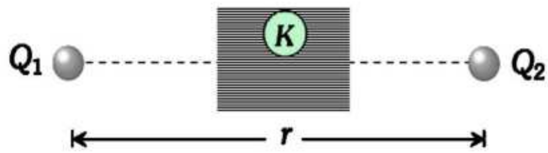
$$F_{medium} = \frac{1}{4\pi\epsilon_0 K} \cdot \frac{Q_1 Q_2}{r^2}$$



(b) If a dielectric medium (dielectric Const. K and thickness t) is partially filled between the charges then effective air separation between the charges becomes $(r - t + t\sqrt{K})$

Hence Force is given by

$$F_{medium} = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{(r - t + t\sqrt{K})^2}$$



Dielectric Constant (K or ϵ_r) -

It is the ratio of absolute electrical permittivity of the medium to the electrical permittivity of free space.

i.e. $$K \text{ or } \epsilon_r = \frac{\epsilon}{\epsilon_0}$$

Also $$K \text{ or } \epsilon_r = \frac{F_{air}}{F_{medium}}$$

For air $K = 1$

For Metals $K = \infty$

Hence $1 \leq K \leq \infty$

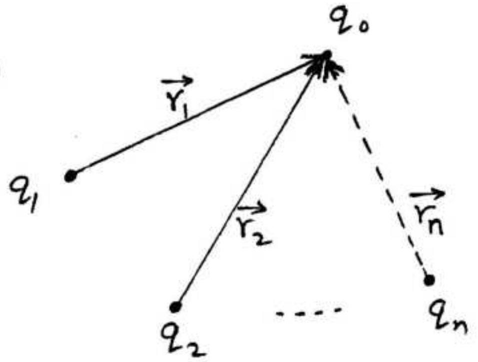
Force between multiple Charges-

According to the principle of superposition, total force acting on a given charge due to number of charges is the vector sum of the individual forces acting on that charge due to all the charges.

$$\vec{F} = \frac{Kq_0q_1}{r_1^2}\hat{r}_1 + \frac{Kq_0q_2}{r_2^2}\hat{r}_2 + \dots \frac{Kq_0q_n}{r_n^2}\hat{r}_n$$

or

$$\boxed{\vec{F} = Kq_0\sum_{i=1}^{n}\frac{q_i}{r_i^2}\hat{r}_i}$$



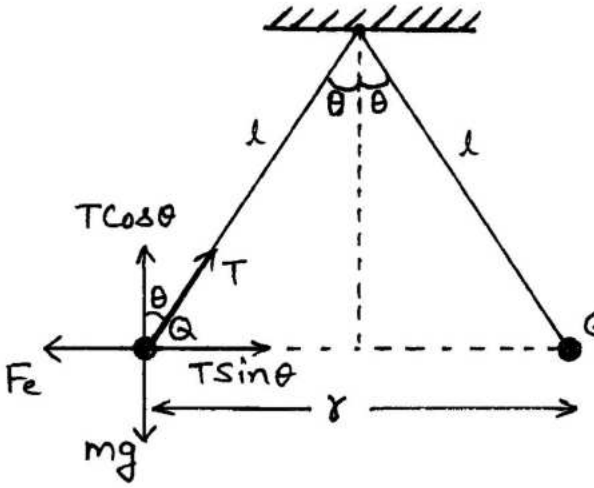
Equilibrium of suspended point charge System-

For equilibrium position

$$T\cos\theta = mg$$

$$T\sin\theta = F_e = \frac{KQ^2}{r^2}$$

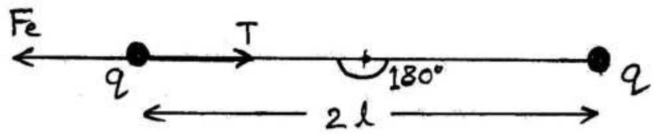
or $$\tan\theta = \frac{F_e}{mg} = \frac{KQ^2}{r^2.mg}$$



Note-

If whole setup is taken into an satellite (where $g \simeq 0$) then

$$T = F_e = \frac{Kq^2}{4l^2}$$

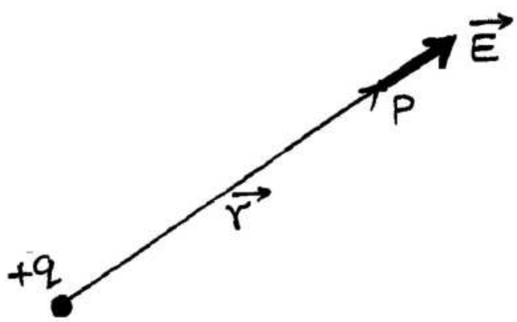


Electric Field -

The region surrounding a charge in which its electric effects can be felt is called the electric field of the given charge.

Electric field intensity due to a point Charge -

It is defined as the force experienced by unit positive charge placed at that point.

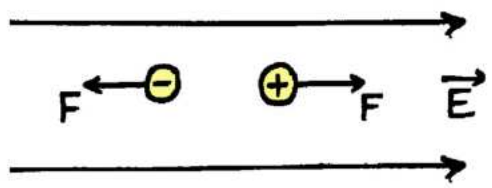


$$\boxed{\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0} \Rightarrow \vec{E} = \frac{Kq}{r^2}\hat{r}}$$

* The electric field is radially outword from a Positive charge and radially inward towards a negative charge.

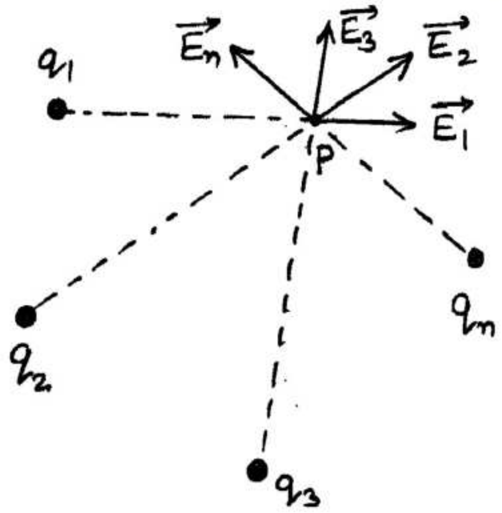
SI Unit - N/c , V/m	Dimensions - $[MLT^{-3}A^{-1}]$

Note - The direction of force on positive charge is along the direction of electric field and on the negative charge is opposite to the electric field.



Electric field intensity due to a system of charges-

By Principle of superposition

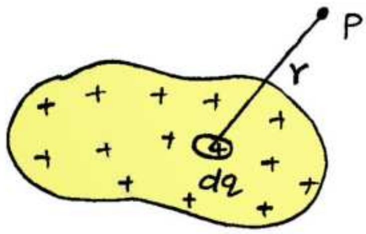


Net electric field $\vec{E} = \vec{E_1} + \vec{E_2} + \ldots \vec{E_n}$

or $$\boxed{\vec{E} = K \sum_{i=1}^{n} \frac{q}{r_i^2}\hat{r}_i}$$

For Continuous charge distribution

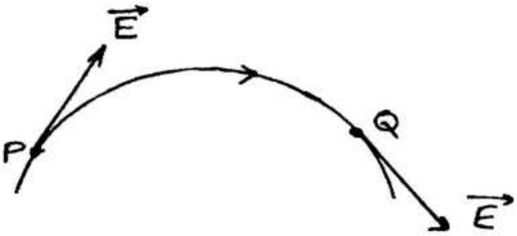
$$\boxed{\vec{E} = K \int \frac{dq}{r^2}\hat{r}}$$



Electric Lines of force -

An electric field line is a curve drawn in such a way that the tangent to it at each point is in the direction of the net electric field at that point.

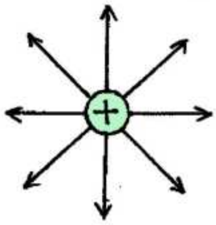
The magnitude of electric field is represented by the density of field lines.

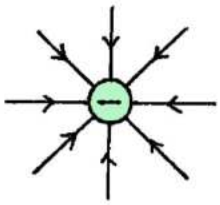


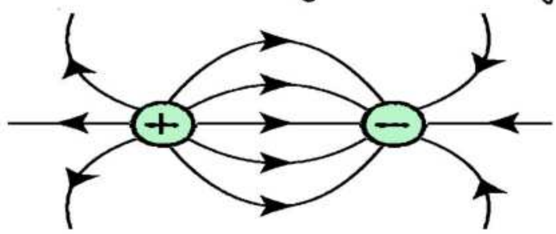
Properties of electric field lines -

1.| Imaginary & Continuous curves.

2.| Starts from positive charge and end at negative charge.







3.| Do not form continuous closed loops.

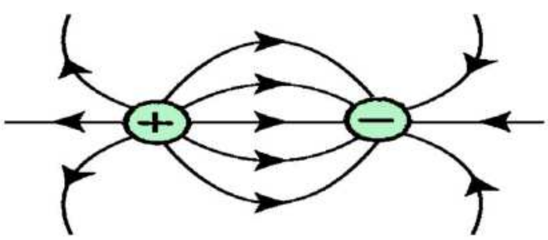
4.| Always normal to the surface of the conductor.

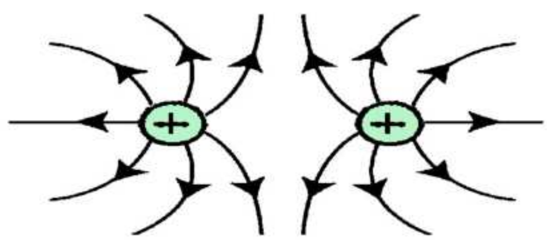
5.| Can never cross each other.

6.| Lines of force per unit area passing normally at a point represents magnitude of electric field.

7.| Around a system of two positive charges field lines gives pictorial representation of their mutual repulsion.

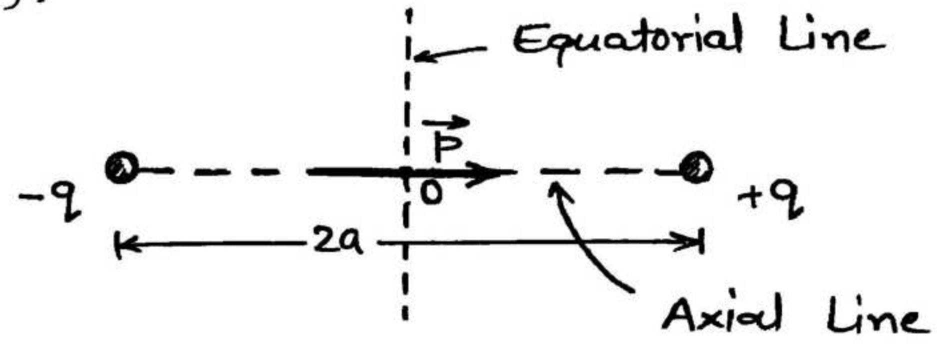
8.| Around a system of two negative charges field lines shows the mutual attraction.





Electric Dipole-

It is a pair of two equal and opposite charges (q and −q) separated by a short distance (2a).



Dipole Moment ($\vec{P}$)-

It is equal to the product of the magnitude of either charge (q) and the distance between them (2a).

$$\vec{P} = q \times \vec{2a}$$

It is a vector quantity whose direction is from −ve charge to +ve charge.

Dipole's Electric field-

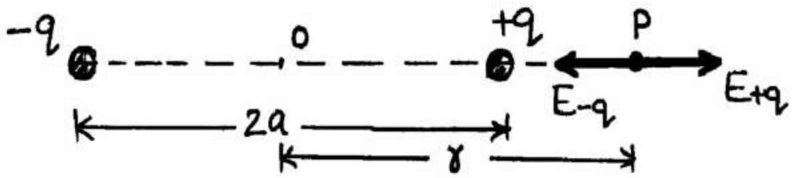
(A.) On axial line -

Electric field at P due to positive charge $E_{+q} = \dfrac{Kq}{(r-a)^2}$

Electric field at P due to negative charge $E_{-q} = \dfrac{Kq}{(r+a)^2}$

Net Electric field E at P

$$E = E_{+q} - E_{-q}$$



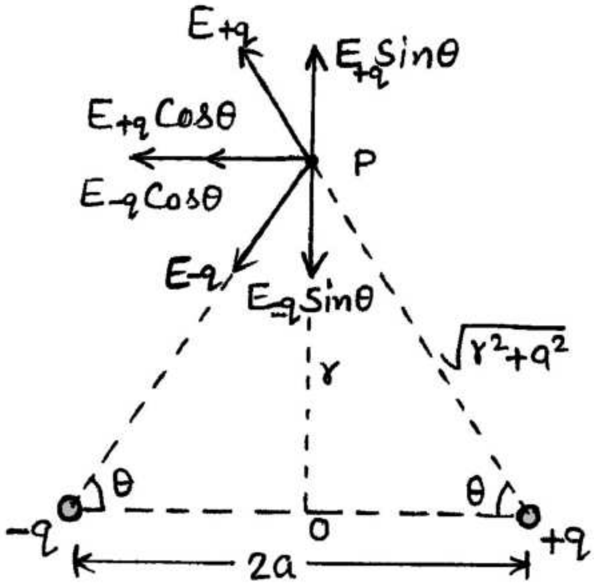
$$E = \frac{Kq}{(r-a)^2} - \frac{Kq}{(r+a)^2}$$

$$E = \frac{Kq \times 4ra}{(r^2-a^2)^2}$$

$$E = \frac{2Kpr}{(r^2-a^2)^2} \qquad [\because p = q \times 2a]$$

If $r >> a$ then $E = \frac{2Kp}{r^3} \Rightarrow \boxed{\vec{E} = \frac{2K\vec{p}}{r^3}}$

(B.) On Equatorial Line -



Electric Field due to positive charge $E_{+q} = \frac{Kq}{(r^2+a^2)}$

Electric Field due to negative charge $E_{-q} = \frac{Kq}{(r^2+a^2)}$

Verticle Component of E_{+q} and E_{-q} will cancel each other and horizontal components will be added.

So net electric field at P is $E = E_{+q}\cos\theta + E_{-q}\cos\theta$

$$E = 2E_{+q}\cos\theta \qquad [\because E_{+q} = E_{-q}]$$

$$E = \frac{2Kq\cos\theta}{(r^2+a^2)} = \frac{2Kqa}{(r^2+a^2)^{3/2}} \qquad \left[\because \cos\theta = \frac{l}{\sqrt{r^2+a^2}}\right]$$

If $r >> a$ then $E = \frac{Kp}{r^3}$

or $\boxed{\vec{E} = \frac{-K\vec{p}}{r^3}}$

Note- $\boxed{E_{axial} = 2E_{equatorial}}$

Dipole in a uniform Electric Field -

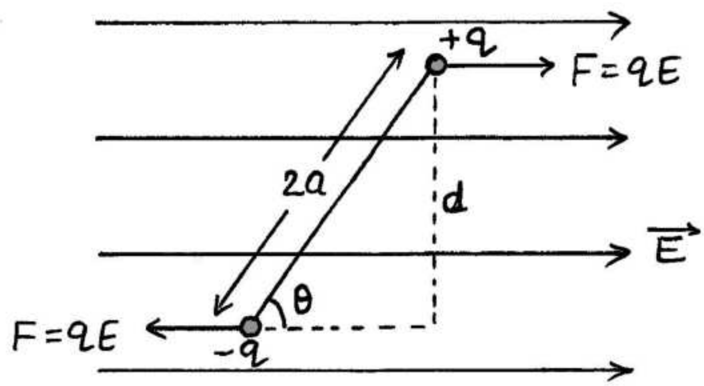
Net Force $F_{net} = 0$

Torque acting on dipole

$\tau = F \times d$

$\tau = qE \times 2a\sin\theta$

$$\boxed{\tau = pE\sin\theta \qquad \vec{\tau} = \vec{p} \times \vec{E}}$$



Case 1. When $\vec{p}$ is along $\vec{E}$, $\theta = 0$ then $\tau = 0$
Dipole is in **stable equilibrium.**

Case 2. When $\vec{p}$ is opposite $\vec{E}$, $\theta = 180^\circ$ then $\tau = 0$
Dipole is in **unstable Equilibrium.**

> **Note** - If $\vec{E}$ is nonuniform then the dipole will experience both a resultant force and a torque, so its motion will be combined translatory and rotatory.

Oscillation of a Dipole in a Uniform $\vec{E}$ -

When a dipole is suspended in a uniform $\vec{E}$, it will align itself parallel to the field. Now if it is given a small angular displacement θ about its equilibrium position then the restoring torque will be $\tau = pE\sin\theta$

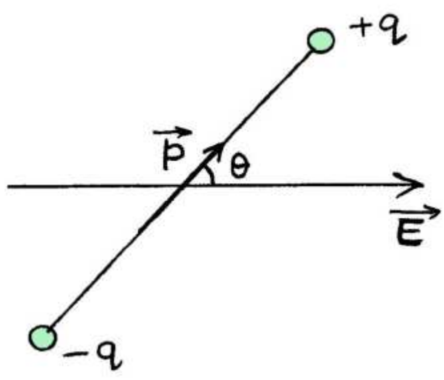
If θ is small then $\tau = pE\theta$

But $\tau = I\alpha$ so that

$I\alpha = pE\theta \Rightarrow \alpha = \frac{pE\theta}{I}$

or $\frac{pE\theta}{I} = \omega^2\theta$ [Angular S.H.M.]

then $\omega = \sqrt{\frac{pE}{I}} \Rightarrow \boxed{T = 2\pi\sqrt{\frac{I}{pE}}}$

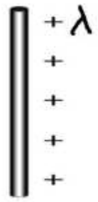


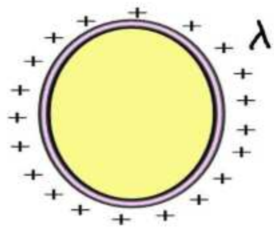
Continuous distribution of charge

1.) Linear Charge distribution -

Linear charge density $\boxed{\lambda = \frac{Q}{l}}$

Example - Charged wire, ring etc.

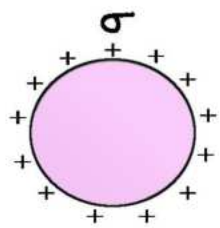


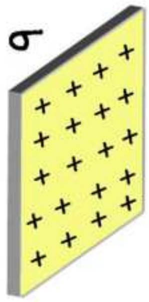


2.) Surface Charge distribution -

Surface charge density $\boxed{\sigma = \frac{Q}{A}}$

Example - Charged plane sheet, sphere etc.

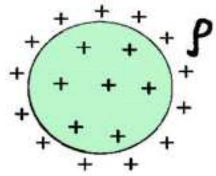




3.) Volume Charge distribution -

Volume charge density $\boxed{\rho = \frac{Q}{V}}$

Example - charged insulating sphere etc.

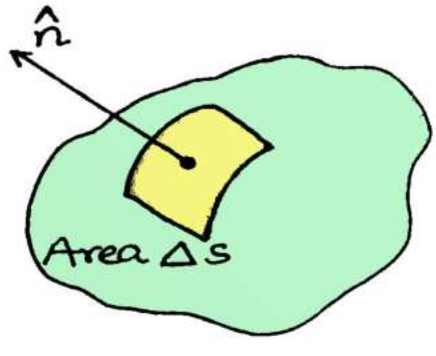


Area Vector -

By convention, the vector associated with every area element of a closed surface is taken to be in the direction of the outward normal.

$$\boxed{\Delta\vec{S} = (\Delta S)\hat{n}}$$

Here ΔS is magnitude of the area element and $\hat{n}$ is a unit vector in the direction of outward normal at that point.



Electric flux (ϕ)-

It is a measure of flow of electric field through a surface. It is equal to the product of an area element and the perpendicular component of electric field ($\vec{E}$) integrated over a surface.

For open surface
- $\phi = \vec{E}\cdot\vec{A}$ (for plane surface)
- $\phi = \int \vec{E}\cdot\vec{ds}$ (for nonuniform surface)

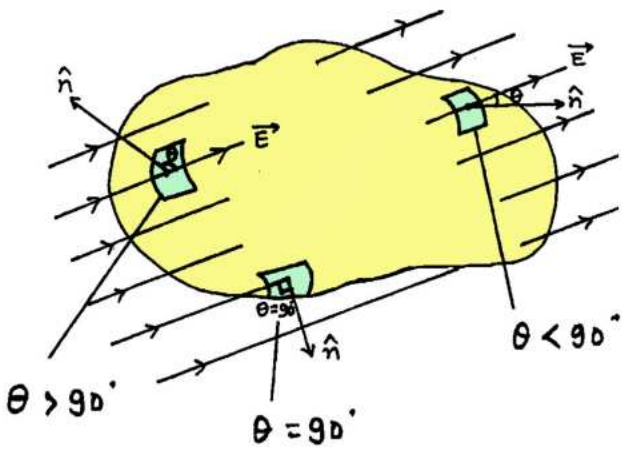
For Closed Surface
- $\phi = \oint \vec{E}\cdot\vec{ds}$ (by definition)
- $\phi = \frac{q}{\epsilon_0}$ (by Gauss's Law)

SI Unit - V.m. or Nm^2/C

Dimensions - $[M^1 L^3 T^{-3} A^{-1}]$

Note- The value of ϕ does not depend upon the distribution of charges and the distance between them inside the closed surface.

For a closed surface outward flux is +ve and inward flux is -ve.



Gauss's Law –

The total flux linked with a closed surface is $\frac{1}{\epsilon_0}$ times the charge enclosed by the closed surface.

$$\boxed{\phi = \oint \vec{E} \cdot \vec{ds} = \frac{q}{\epsilon_0}}$$

Important facts about Gauss's Law –

(i) Gauss's law is valid only for the fields, which follow coulomb's 'inverse square law'.

(ii) According to Gauss's Law, electric flux passing through a closed surface doesn't depend on shape of surface.

(iii) Electric flux passing through a closed surface doesn't depends on charge distribution or location of charge inside the surface.

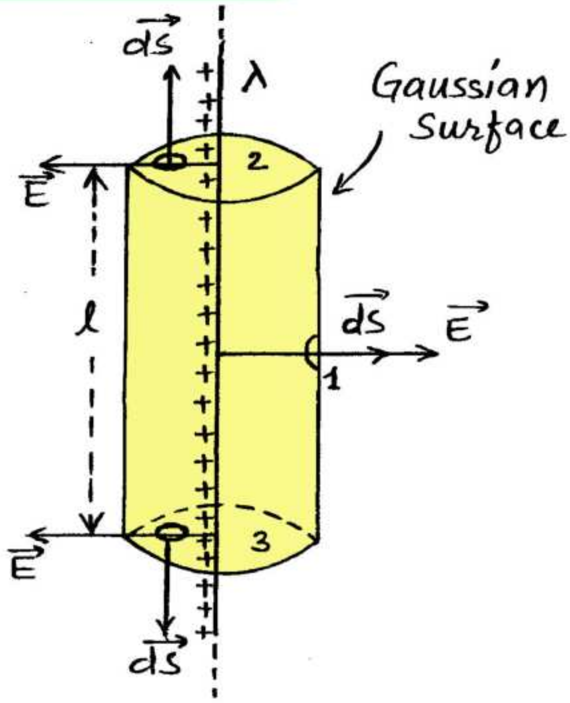
(iv) Net electric flux depends on value and nature of the charge and medium.

(v) If a symmetric body has n identical faces and charge q is located at its centre then electric flux associated with each of its face is $\phi/n = q/n\epsilon_0$

(vi) Net electric flux associated with a closed surface placed in an electric field (uniform or non uniform) is zero, if there is no charge inside the surface.

Electric Field due to special Charge distribution-

1.) An infinite Line distribution of charge-



From Gauss's theorem

$$\oint \vec{E}\cdot\vec{ds} = \frac{q}{\epsilon_0}$$

$$\int_1 \vec{E}\cdot\vec{ds} + \int_2 \vec{E}\cdot\vec{ds} + \int_3 \vec{E}\cdot\vec{ds} = \frac{q}{\epsilon_0}$$

$$\int_1 E ds + 0 + 0 = \frac{q}{\epsilon_0}$$

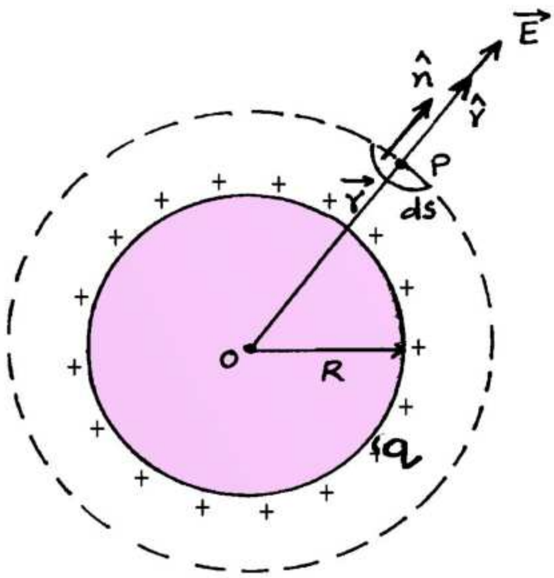
$$E \times 2\pi r l = \frac{q}{\epsilon_0}$$

$$E \times 2\pi r l = \frac{\lambda l}{\epsilon_0}$$

or $E = \dfrac{\lambda}{2\pi\epsilon_0 r} \Rightarrow \boxed{E = \dfrac{2K\lambda}{r}}$

2.) An Conducting or Hollow sphere -

(i) For Outside point -



Using Gauss's theorem

$$\oint \vec{E}\cdot\vec{ds} = \frac{q}{\epsilon_0}$$

$$\oint E ds \cos 0^\circ = \frac{q}{\epsilon_0}$$

$$E \oint ds = \frac{q}{\epsilon_0}$$

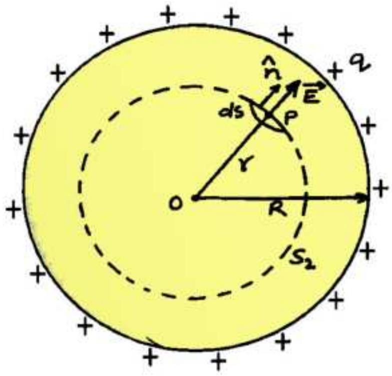
$$E \times 4\pi r^2 = \frac{q}{\epsilon_0}$$

$E = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r^2} \Rightarrow \boxed{E = \dfrac{Kq}{r^2}}$

(ii) For surface point $r = R$

$E = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{R^2} \Rightarrow \boxed{E = \dfrac{Kq}{R^2}}$

(iii) For Inside point ($r < R$)



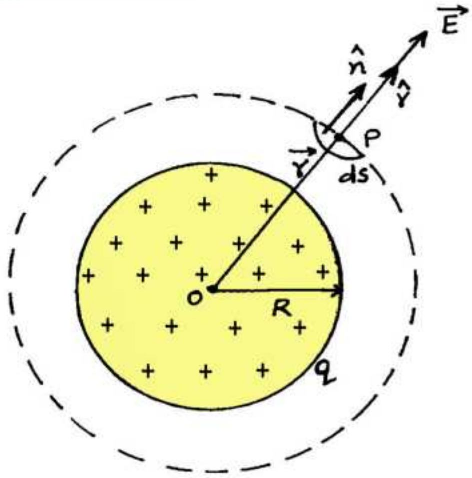
Because charge inside the conducting or hollow sphere is zero.

$$\oint \vec{E}\cdot\vec{ds} = \frac{q}{\epsilon_0} = 0$$

Hence $\boxed{E = 0}$

3.] An solid nonconducting sphere-

(i) Outside Point ($r > R$)-



From Gauss's theorem

$$\oint \vec{E}\cdot\vec{ds} = \frac{q}{\epsilon_0}$$

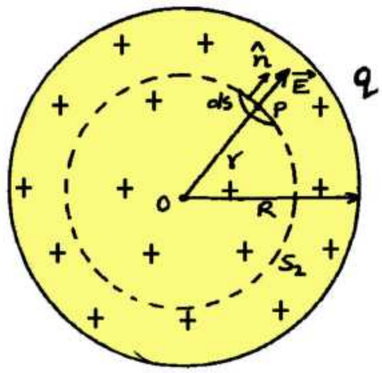
$$E \times 4\pi r^2 = \frac{q}{\epsilon_0}$$

or $$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} \Rightarrow \boxed{E = \frac{Kq}{r^2}}$$

(ii) At surface ($r = R$)-

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2} \Rightarrow \boxed{E = \frac{Kq}{R^2}}$$

(iii) Inside Point ($r < R$)-



From Gauss's theorem

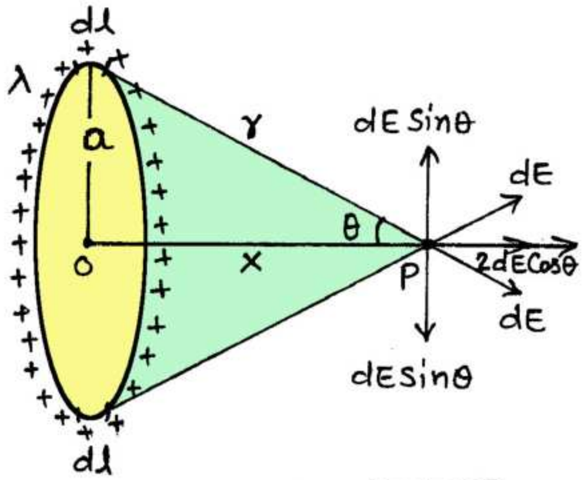
$$\oint \vec{E}\cdot\vec{ds} = \frac{q}{\epsilon_0}$$

$$E \times 4\pi r^2 = \frac{1}{\epsilon_0}\times\left(\frac{q r^3}{R^3}\right)$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{q r}{R^3} \Rightarrow \boxed{E = \frac{Kqr}{R^3}}$$

4.] At the axis of a charged Ring -

Let a small element dl have charge $dq = \lambda \times dl = \frac{q}{2\pi a} \times dl$



$r = \sqrt{a^2+x^2}$

Electric field due to this charge at P, $dE = \frac{dq}{4\pi\epsilon_0 r^2}$

An another diametrically opposite element of length dl also produces electric field dE.

The vertical components will cancel each other and horizontal components will be added.

So net electric field $E = \sum dE \cos\theta = \int dE \cos\theta$

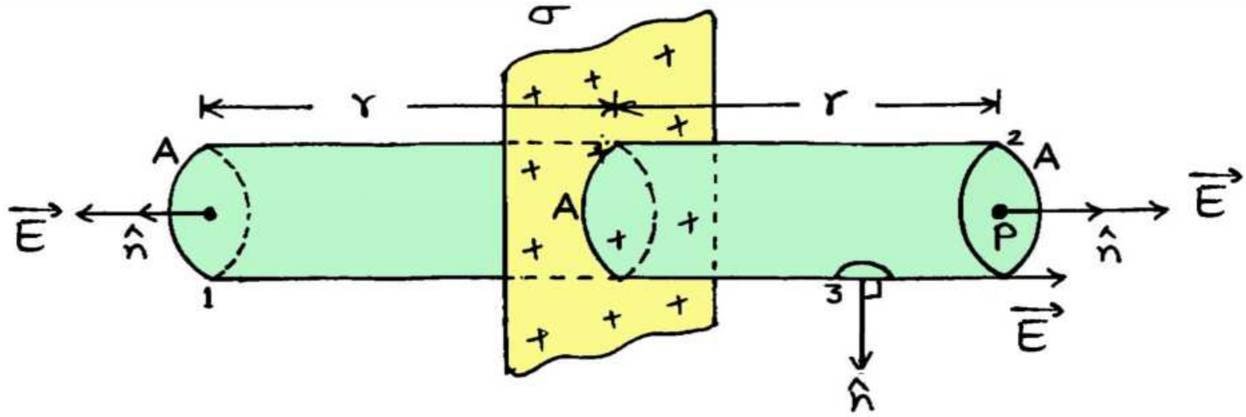
$$E = \int \frac{dq}{4\pi\epsilon_0 r^2} \times \frac{x}{r} \Rightarrow E = \int \frac{q\,dl}{2\pi a} \times \frac{1}{4\pi\epsilon_0 r^3} \times \frac{x}{r}$$

$$E = \frac{q}{2\pi a} \cdot \frac{x}{4\pi\epsilon_0 r^3} \int dl \Rightarrow E = \frac{q}{2\pi a} \times \frac{x}{4\pi\epsilon_0 r^3} \times 2\pi a$$

$$E = \frac{qx}{4\pi\epsilon_0 r^3} \Rightarrow \boxed{E = \frac{qx}{4\pi\epsilon_0 (a^2+x^2)^{3/2}} = \frac{Kqx}{(a^2+x^2)^{3/2}}}$$

* At centre $(x=0)$ $E = 0$
* If x increases Electric field E due to ring first ↑ then ↓ and at $x = \frac{a}{\sqrt{2}}$ it is maximum.

5.] An infinite plane sheet of charge (non- Conducting) -



By Gauss's theorem $\oint \vec{E} \cdot \vec{ds} = \frac{q}{\epsilon_0}$

$$\int_1 \vec{E} \cdot \vec{ds} + \int_2 \vec{E} \cdot \vec{ds} + \int_3 \vec{E} \cdot \vec{ds} = \frac{q}{\epsilon_0}$$

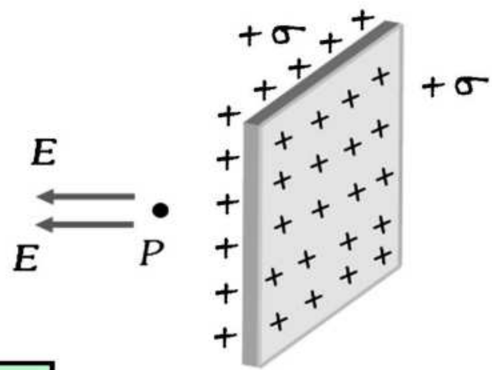
$$\int_1 E ds + \int_2 E ds + 0 = \frac{q}{\epsilon_0}$$

$$E \int_1 ds + E \int_2 ds = \frac{q}{\epsilon_0}$$

$$EA + EA = \frac{q}{\epsilon_0} = \frac{\sigma A}{\epsilon_0} \Rightarrow \boxed{E = \frac{\sigma}{2\epsilon_0}}$$

6.] An infinite conducting plane sheet of Charge -

When a charge is given to a Conducting plate, it distributes itself over the entire surface of the plate. The surface density σ is uniform and is the same on both surfaces.



Hence $E = 2 \times \frac{\sigma}{2\epsilon_0} \Rightarrow \boxed{E = \frac{\sigma}{\epsilon_0}}$

Equilibrium of suspended charge from string -

In equilibrium -

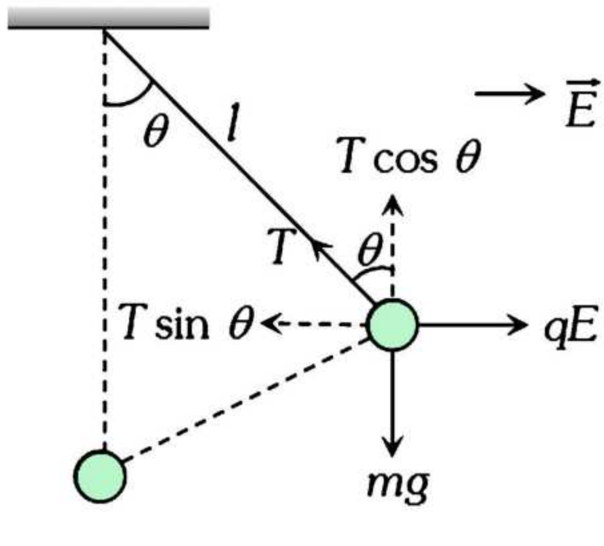
$T\sin\theta = qE$ — ①

$T\cos\theta = mg$ — ②

From eq. ① and ②

$T = \sqrt{(qE)^2 + (mg)^2}$

and $\tan\theta = \dfrac{qE}{mg}$



Equilibrium of charged Soap Bubble -

The air pressure inside the soap bubble is $4T/r$ more than the atmospheric pressure.

Pressure excess $P = \dfrac{4T}{r}$

Now if the surface charge density on charging the soap bubble is σ then electric pressure on this charged bubble towards the outer side of the surface will be $\dfrac{\sigma^2}{2\epsilon_0}$.

When the bubble is charged then a condition comes when the pressure excess becomes zero. In this condition the charged bubble bursts.

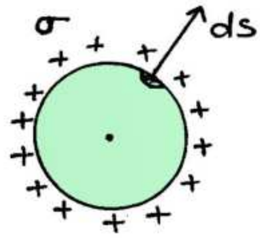
Now in equilibrium state $\dfrac{4T}{r} = \dfrac{\sigma^2}{2\epsilon_0}$

Radius of the bubble $\boxed{r = \dfrac{8T\epsilon_0}{\sigma^2}}$

Electrostatic Pressure -

The electric force acting per unit area of charged surface is defined as electrostatic pressure.

Force on small element ds of charged conductor dF = (charge on ds) × Electric field



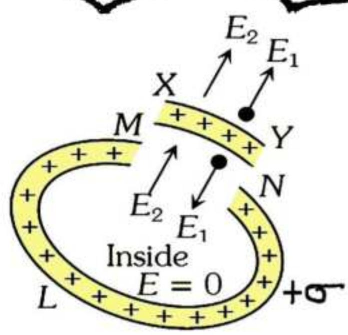
$$dF = (\sigma\, ds) \times \frac{\sigma}{2\epsilon_0} \Rightarrow dF = \frac{\sigma^2}{2\epsilon_0} ds$$

Hence $\boxed{P_{ele.} = \frac{dF}{ds} = \frac{\sigma^2}{2\epsilon_0}}$ (Normally Outwards)

Inside $E_1 - E_2 = 0 \Rightarrow E_1 = E_2$

Outside $E_1 + E_2 = E \Rightarrow E = 2E_2$

$$\boxed{E_2 = \frac{E}{2} = \frac{\sigma}{2\epsilon_0}}$$



Electrostatic Potential Energy -

The electrostatic potential energy of a charge q at a point in an electric field is equal to the work done by the external force in bringing the charge q from infinity to that point without any acceleration.

$$U_P = W_{\infty P} = \int_{\infty}^{P} \vec{F}_{ext} \cdot \vec{dr} = -\int_{\infty}^{P} \vec{F_e} \cdot \vec{dr}$$

Also electrostatic potential energy difference between two points A and B is equal to the minimum work done by an external force to move a charge q from A to B without acceleration.

$$\Delta U = U_B - U_A = W_{AB} = \int_{A}^{B} \vec{F}_{ext} \cdot \vec{dr} = -\int_{A}^{B} \vec{F_e} \cdot \vec{dr}$$

Note - The work done by electric field in moving the charge does not depend on the path.

Electrostatic Potential -

Potential at a point in a field is defined as the amount of work done in bringing a unit positive test charge from infinity to that point.

$$V_P = \frac{W_{\infty P}}{q_0} \Rightarrow V_P = \frac{U_P}{q_0}$$

Also electric potential difference between two points A and B in an electric field is equal to the work done in moving unit positive test charge from A to B.

$$\Delta V = V_B - V_A = \frac{W_{AB}}{q_0} = \frac{U_B - U_A}{q_0}$$

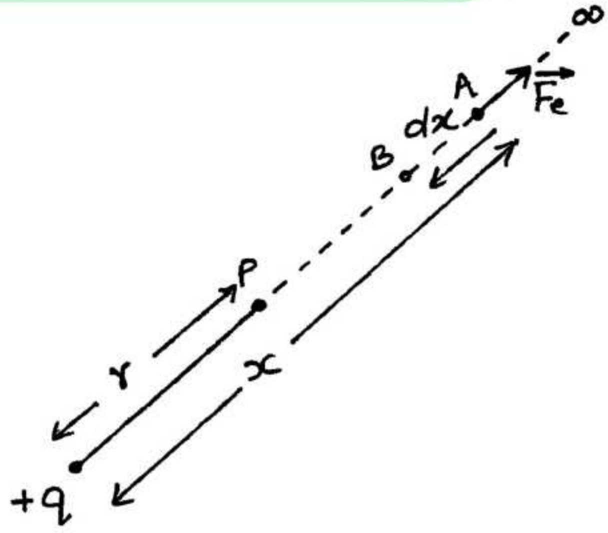
* Electrostatic Potential is a scalar quantity.

SI Unit - Joule/Coulomb or Volt   Dimensions - $[M^1L^2T^{-3}A^{-1}]$

Electrostatic Potential due to a Point Charge-

Force on test charge q_0 due to source charge q at point A is

$$F_e = \frac{Kqq_0}{x^2}$$



Now small work done in small displacement dx is

$$dw = \vec{F_e}\cdot\vec{dx} = F_e dx \cos 180°$$

$$dw = -F_e dx$$

Total work done to move the charge from ∞ to point P is

$$W = \int_{\infty}^{r} -F_e dx = -\int_{\infty}^{r} \frac{Kqq_0}{x^2} dx$$

or $$W = -Kqq_0 \int_{\infty}^{r} x^{-2} dx = -Kqq_0 \left[-\frac{1}{x}\right]_{\infty}^{r}$$

$$W = Kqq_0\left[\frac{1}{r} - \frac{1}{\infty}\right] \Rightarrow W = \frac{Kqq_0}{r}$$

By definition $V = \frac{W}{q_0}$ $\Rightarrow$ $\boxed{V = \frac{Kq}{r}}$

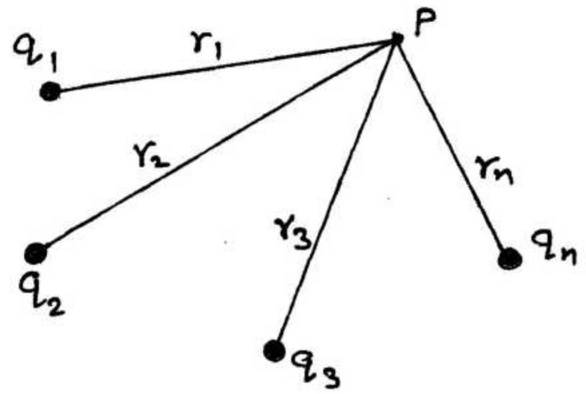
★ If q is positive then V is positive and when q is negative then V is negative.

Potential at a point due to group of electric charges-

By superposition Principle
Net electric potential at P is

$$V = V_1 + V_2 + \dots V_n$$

$$\boxed{V = K\sum_{i=1}^{n} \frac{q_i}{r_i}}$$



Electric Potential at a point due to an Electric Dipole

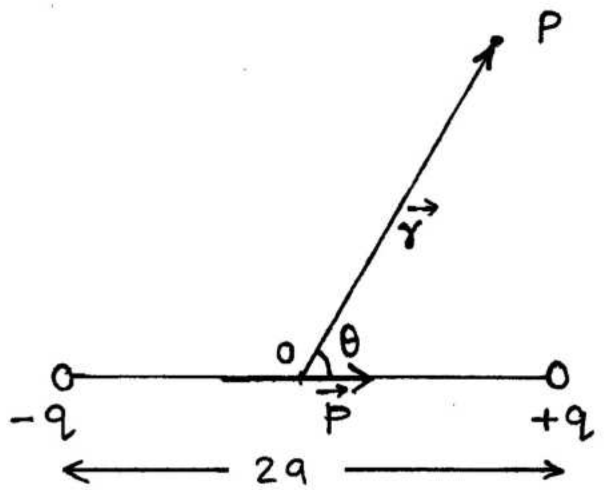
(A.) At general point -

$$E = \frac{KP}{r^3}\sqrt{3\cos^2\theta + 1}$$

$$V = \frac{KP\cos\theta}{r^2}$$

* Angle between $\vec{E}$ and $\vec{P}$ is $(\theta + \alpha)$

Where $\alpha = \tan^{-1}\left(\frac{1}{2}\tan\theta\right)$



(B.) At Axial point -

$$\vec{E} = \frac{K\vec{P}}{r^3}$$

$$V = \frac{KP}{r^2}$$

(C.) At Equatorial point -

$$\vec{E} = -\frac{K\vec{P}}{r^3} \quad \text{and} \quad V = 0$$

Potential Energy of a dipole in an external Field -

Work done in rotating an electric dipole by $d\theta$ is

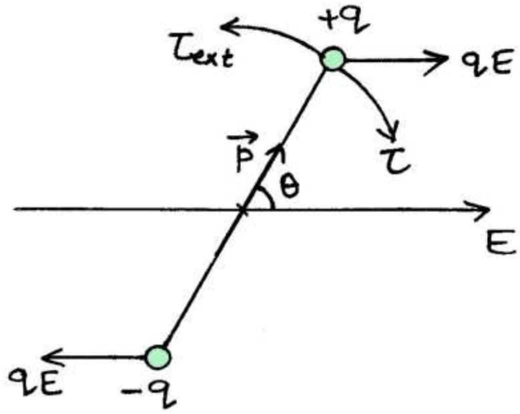
$$dW = \tau_{ext}\, d\theta$$

Total work done to rotate the dipole from θ_1 to θ_2 is

$$W = \int_{\theta_1}^{\theta_2} dW = \int_{\theta_1}^{\theta_2} \tau_{ext}\, d\theta$$

$$W = \int_{\theta_1}^{\theta_2} PE\sin\theta\, d\theta = -PE\left[\cos\theta\right]_{\theta_1}^{\theta_2}$$

$$W = -PE\left[\cos\theta_2 - \cos\theta_1\right]$$



$$\boxed{\tau_{ext} \simeq \tau}$$

or $$\boxed{\Delta U = U_2 - U_1 = W = -PE\left[\cos\theta_2 - \cos\theta_1\right]}$$

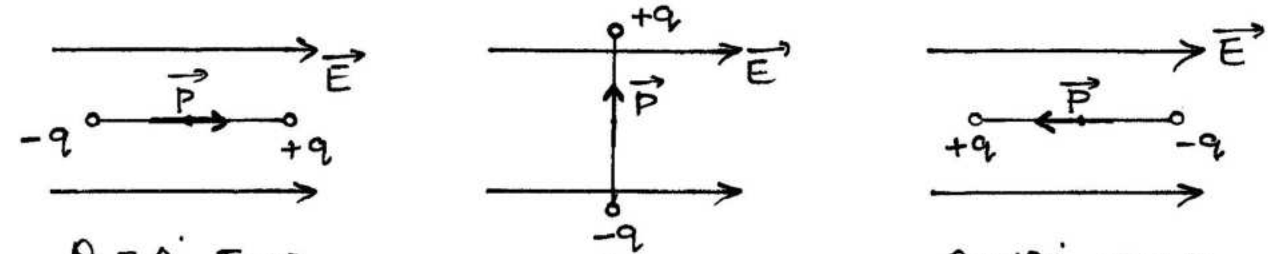
Note -

(i) Electrostatic potential energy of a dipole in uniform electric field is defined as work done in rotating a dipole from a direction perpendicular to the field to the given direction.

$$U = W_\theta - W_{90^\circ} = pE\left[\cos 90^\circ - \cos\theta\right]$$

$$\boxed{U = -pE\cos\theta = -\vec{p}\cdot\vec{E}}$$

(ii)



$\theta = 0^\circ$, $\tau = 0$
$U_{min} = -pE$
Stable equilibrium

$\theta = 90^\circ$, $\tau_{max} = pE$
$U = 0$

$\theta = 180^\circ$, $\tau = 0$
$U_{max} = pE$
Unstable equilibrium

Equipotential Surfaces-

A surface on which the electric potential is same at every point is called equipotential surface.

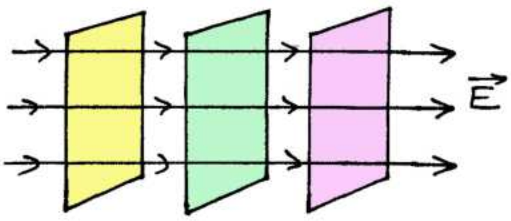
Properties of equipotential surfaces-

1.) No work is done on moving the test charge from one point to another on equipotential surface.

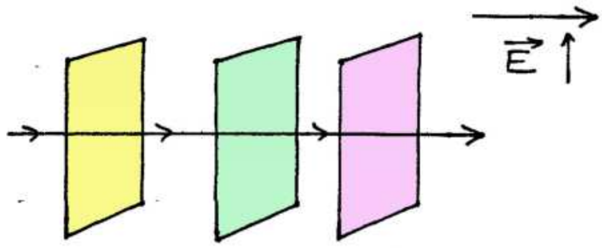
2.) The equipotential surface through a point is normal to the electric field at that point.

3.) For uniform electric field the equipotential surfaces are equally spaced parallel planes perpendicular to the field lines.

4.) In a region where $\vec{E}$ is strong the equipotential surfaces are close together and where $\vec{E}$ is weaker, the equipotential surfaces are farther apart.

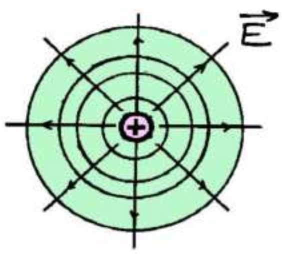


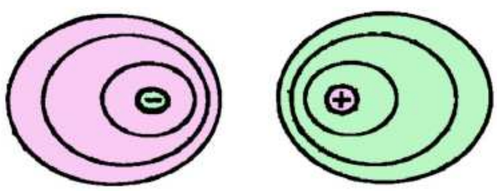
Uniform $\vec{E}$



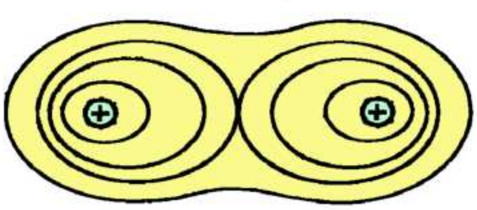
Non uniform $\vec{E}$

5.) For a point charge equipotential surfaces are concentric spheres.





For an electric dipole.



For two identical Positive charges

6.) Equipotential surfaces never intersect each other.

Relation between Electric field and Electric potential

The magnitude of electric field is given by the change in magnitude of potential per unit displacement normal to the equipotential surface at the point.

$$\boxed{E = -\frac{dV}{dr}} = -(\text{Potential Gradient})$$

Here the negative sign indicates that in the direction of electric field, potential decreases.

Special Points-

1.| Potential is a scalar quantity but the gradient of potential is a vector quantity.

2.| Direction of $\vec{E}$ is from high potential to low potential.

3.| In case of cartesian coordinates -

$$\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$$

where $E_x = -\frac{\partial V}{\partial x}$, when y and z are taken Constant.

$E_y = -\frac{\partial V}{\partial y}$, where x and z are taken Constant.

$E_z = -\frac{\partial V}{\partial z}$, where x and y are taken constant.

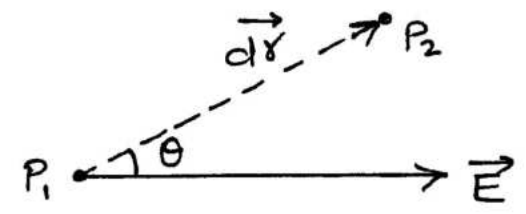
4.| By formula $E = -dV/dr$, potential difference between any two points in an electric field can be given as

$$\boxed{dV = -\vec{E}\cdot\vec{dr} \quad \Rightarrow \quad \Delta V = -\int_{r_1}^{r_2}\vec{E}\cdot\vec{dr}}$$

5.| If at any point $E = 0$, then V = Constant (zero or non zero) because $E = -\frac{dV}{dr}$.

6.) From $dV = -\vec{E}\cdot\vec{dr}$

$= -E\,dr\cos\theta$



We get, $-\frac{dV}{dr} = E\cos\theta$

Thus $-\frac{dV}{dr}$ gives the component of the electric field in the direction of $\vec{dr}$.

Case 1: If $\vec{dr}$ is in the direction of $\vec{E}$, θ is zero and hence $-\frac{dV}{dr} = E$ is maximum.

★ So the electric field is along the direction in which the potential decreases at the maximum rate.

Case 2: If $\theta = 90^\circ$, $\cos\theta = 0$

so that $dV = -E\,dr\cos\theta = 0$ or $V = \text{Constant}$

★ Thus the potential remains constant in a direction perpendicular to the electric field. Conversely, the field is perpendicular to the line or plane along which the potential is const.

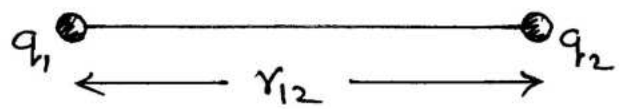
Electrostatic Potential Energy of a system of charges-

$$\boxed{\text{Work done in assembling a system} = U_{system}}$$

1.) For a system of two charges -

$W_1 = 0$

by definition $V = \dfrac{W_2}{q_2}$



$W_2 = q_2 V = q_2 \times \dfrac{Kq_1}{r_{12}}$

Hence $U_{system} = W_{net}$ $\Rightarrow$ $\boxed{U_{system} = \dfrac{Kq_1q_2}{r_{12}}}$

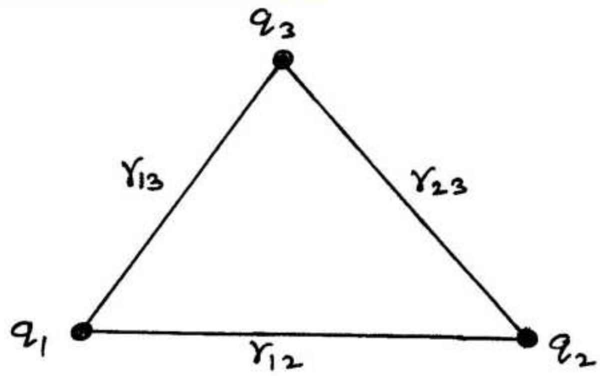
2.) For a system of three or n charges-

$W_1 = 0$

$W_2 = \dfrac{Kq_1q_2}{r_{12}}$

Similarly $W_3 = q_3 V$

$W_3 = q_3\left[\dfrac{Kq_1}{r_{13}} + \dfrac{Kq_2}{r_{23}}\right]$



Hence $U_{system} = W_{net}$

$$\boxed{U_{system} = \frac{Kq_1q_2}{r_{12}} + \frac{Kq_1q_3}{r_{13}} + \frac{Kq_2q_3}{r_{23}}}$$

Similarly for n charge system

$$\boxed{U_{system} = K \sum_{\substack{i \neq j \\ \text{All pairs}}} \frac{q_i q_j}{r_{ij}}}$$

Note - In calculating potential energy, the values of all charges are taken with their given signs. Hence potential energy of a system may be +ve or -ve.

Potential Energy of a system of charges in external field-

1.| P.E. of a single charge-

By definition $V = \frac{W}{q} \Rightarrow W = qV$

Hence $U = W \Rightarrow$ $\boxed{U = qV}$

2.| P.E. of a system of two charges-

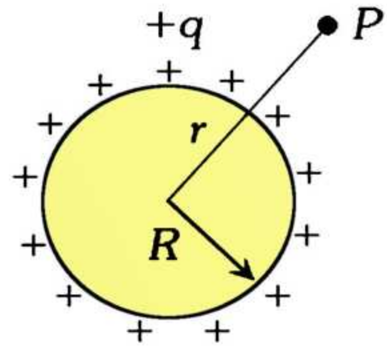
By definition $W_1 = q_1V_1$ and $W_2 = q_2V_2 + \frac{Kq_1q_2}{r_{12}}$

Hence $U_{system} = W_{net} \Rightarrow$ $\boxed{U_{system} = q_1V_1 + q_2V_2 + \frac{Kq_1q_2}{r_{12}}}$

by Umesh Rajoria
SCIENCE CAREER COACHING
Cont. 8003024131, 9309068859

Electric Potential due to hollow or conducting sphere –

(i) At outside sphere $(r > R)$ –



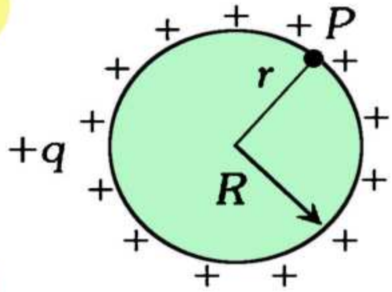
By definition $V = -\int_{\infty}^{r} \vec{E} \cdot \vec{dr}$

$$V = -\int_{\infty}^{r} \frac{q}{4\pi\epsilon_0 r^2} dr \qquad \left[\because E_{out} = \frac{q}{4\pi\epsilon_0 r^2}\right]$$

$$V = -\frac{q}{4\pi\epsilon_0} \int_{\infty}^{r} r^{-2} dr = -\frac{q}{4\pi\epsilon_0} \left[-\frac{1}{r}\right]_{\infty}^{r}$$

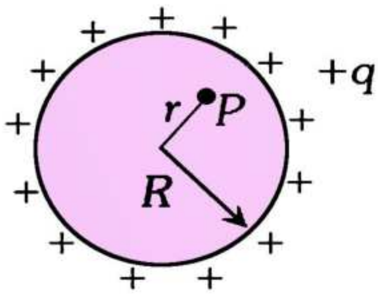
$$V = \frac{-q}{4\pi\epsilon_0} \left[\frac{1}{r} - \frac{1}{\infty}\right] \Rightarrow \boxed{V = \frac{1}{4\pi\epsilon_0} \frac{q}{r} = \frac{Kq}{r}}$$

(ii) At surface of sphere $(r = R)$ –



$$\boxed{V = \frac{1}{4\pi\epsilon_0} \frac{q}{R} = \frac{Kq}{R}}$$

(iii) At inside the sphere $(r < R)$ –



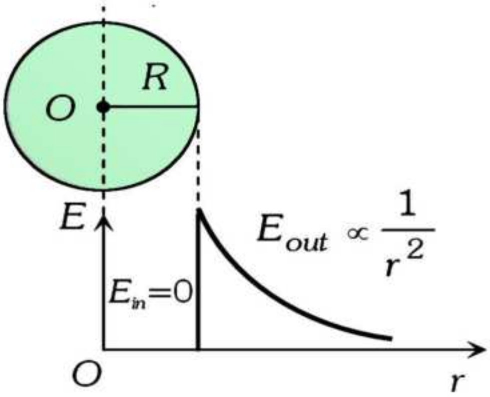
$\because$ Inside the sphere $E = 0$

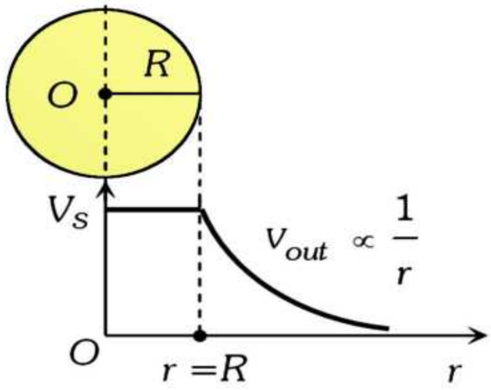
and $-\frac{dV}{dr} = E \Rightarrow \frac{dV}{dr} = 0$

or $V = \text{Const}$

so $\boxed{V = \frac{1}{4\pi\epsilon_0} \frac{q}{R} = \frac{Kq}{R}}$

Graph

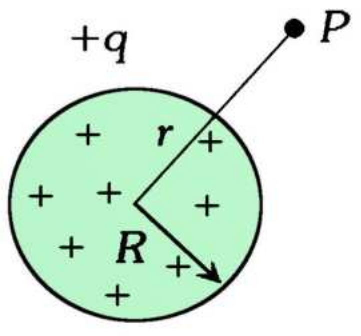




Electric Potential due to solid Non-Conducting Sphere-

(i) At outside sphere ($r > R$) -

$$\boxed{V = \frac{Kq}{r}}$$

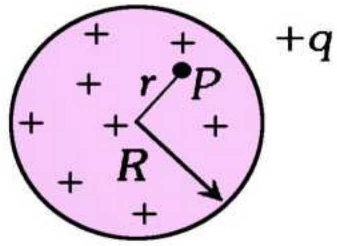


(ii) At surface of sphere ($r = R$) -

$$\boxed{V = \frac{Kq}{R}}$$

(iii) At inside the sphere ($r < R$) -

By definition $V = -\int_{\infty}^{r} \vec{E}\cdot\vec{dr}$



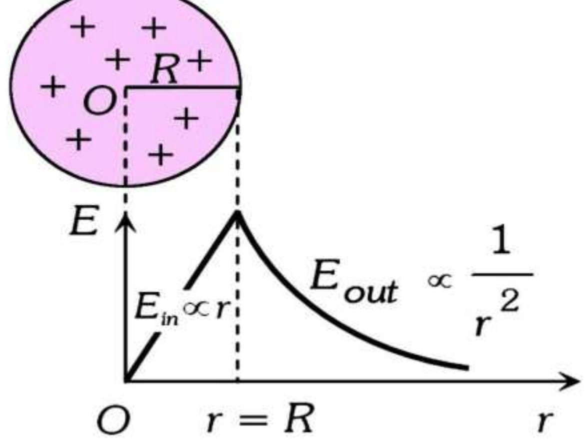
$$\Rightarrow V = -\left[\int_{\infty}^{R} E_1 dr + \int_{R}^{r} E_2 dr\right]$$

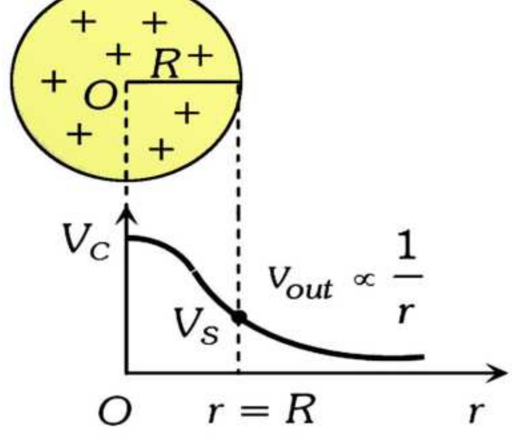
$$\Rightarrow V = -\left[\int_{\infty}^{R} \frac{Kq}{r^2} dr + \int_{R}^{r} \frac{Kqr}{R^3} dr\right]$$

$$V = -\left[Kq\left[\frac{-1}{r}\right]_{\infty}^{R} + \frac{Kq}{R^3}\left[\frac{r^2}{2}\right]_{R}^{r}\right]$$

$$V = -Kq\left[-\frac{1}{R} + \frac{r^2}{2R^3} - \frac{R^2}{2R^3}\right] \Rightarrow \boxed{V = \frac{Kq}{2R^3}\left[3R^2 - r^2\right]}$$

Graph

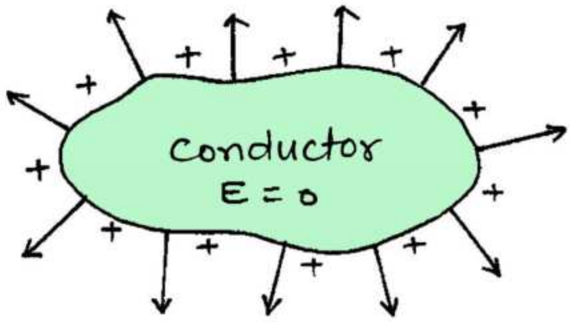




Electrostatics of Conductors -

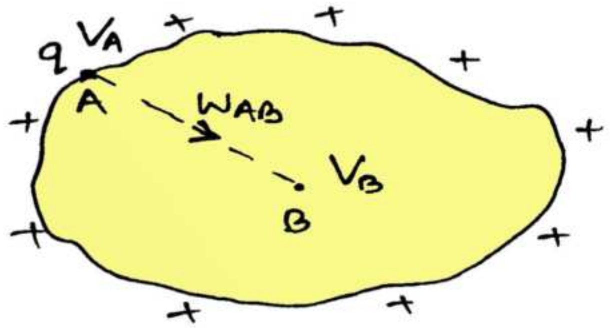
1.| Inside a conductor electric field is zero.

2.| At the surface of a charged conductor, electric field must be normal to the surface at every point.



3.| The interior of a conductor can have no excess charge in the static situation.

4.| Electrostatic potential is constant throughout the volume of the conductor and has the same value (as inside) on its surface.



Here inside the Conductor $E = 0$

by $\frac{W_{AB}}{q} = V_B - V_A$

So that $W_{AB} = 0$

Hence $\frac{0}{q} = V_B - V_A$

$V_B - V_A = 0$

$$\boxed{V_B = V_A}$$

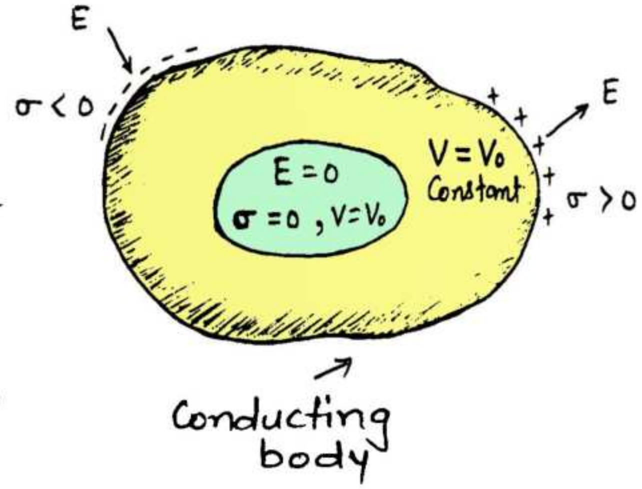
5.| Electric field at the surface of a charged conductor is $\vec{E} = \frac{\sigma}{\epsilon_0}\hat{n}$, where σ is surface charge density which may be +ve or -ve and $\hat{n}$ is the unit vector normal to the surface in the outward direction.

6.| Electrostatic shielding -

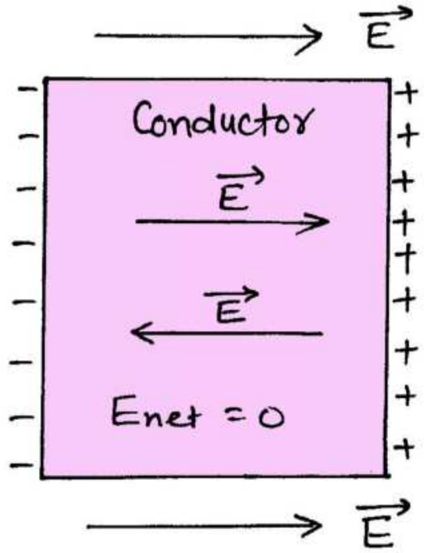
It is the phenomenon of protecting a certain region of space from external electric field.

If a conductor contains a cavity and if no charge is present inside the cavity, then there can be no net charge anywhere on the surface of the cavity.

This means that if you are inside a charged conducting box, you can safely touch any point on the inside walls of the cavity without being electrocuted.

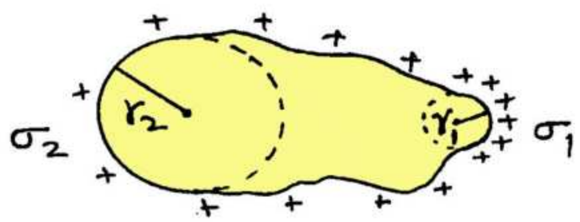


7.| If a uncharged conducting plate is placed in a region having electric field, then due to electric force on the free electrons in the conductor, they redistributes themselves till the electric field produced due to the redistribution cancel the original external field. Hence the net field inside the conductor becomes zero.



8.| A nonuniform charged conductor have nonuniform surface charge density at different points. Surface charge density is maximum where the radius of curvature is minimum. i.e. $\sigma \propto \frac{1}{r}$

$$\boxed{\frac{\sigma_1}{\sigma_2} = \frac{r_2}{r_1}}$$



Dielectrics-

Dielectrics are insulating (non-conducting) materials which transmit electrical effect without conducting.

Dielectrics

(i) Polar Dielectrics

→ Molecules have permanent electric dipole moment ($\vec{P}$)

→ Ex.- HCl, H_2O, NH_3 etc.

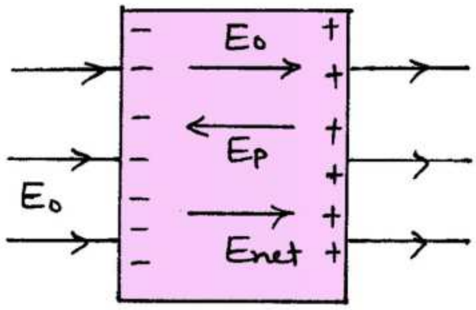
(ii) Non-polar Dielectrics

→ Molecules has zero dipole moment ($\vec{P}$)

→ Ex. - O_2, CO_2, CH_4 etc.

Polarization of dielectric slab-

It is the process of inducing equal and opposite charges on the two faces of the dielectric on the application of electric field.



Net electric field inside the dielectric is given by -

$$\boxed{E_{net} = E_0 - E_p} \quad \text{or} \quad \boxed{E_{net} = \frac{E_0}{K}}$$

$K \rightarrow$ dielectric Const.

Thus the effect of dielectric is to weaken the applied field inside the dielectric.

Note- If a very high electric field is applied in a dielectric it may behave like a conductor due to detachment of the electrons from their parent atoms. This phenomenon is known as the dielectric breakdown.

The maximum field that a dielectric material can tolerate without its electric breakdown is called its dielectric strength.

* The dielectric strength of air is 3×10^6 V/m.

Electric Capacitance

Concept of Capacitance -

When a conductor is charged then its potential rises. The increase in potential is directly proportional to the charge given to the conductor.

$Q \propto V$ or $\boxed{Q = CV}$ or $\boxed{C = \frac{Q}{V}}$

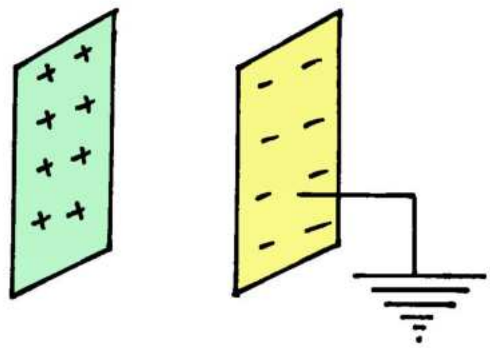
The constant C is known as capacitance of the conductor.

SI Unit - Farad or Coulomb/volt.

Capacitor or Condenser -

It is a device that stores electric energy or a capacitor is a pair of two conductors, separated by an insulator (or a vacuum) and have equal and opposite charge.

A capacitor get's charged when a battery is connected across the plates. Once capacitor get's fully charged, flow of charge stops in the circuit and the potential difference across the plates of capacitor is same as the potential difference across the terminals of battery.

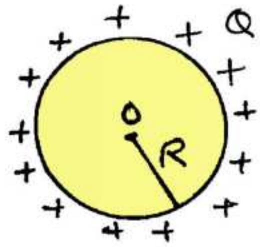


Capacitance of a Spherical Conductor -

Potential $V = \frac{1}{4\pi\epsilon_0}\frac{Q}{R}$

By $C = \frac{Q}{V}$ $\Rightarrow$ $\boxed{C = 4\pi\epsilon_0 R}$

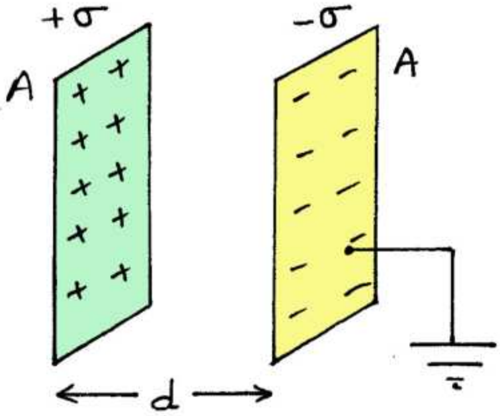
In a medium $\boxed{C_{medium} = 4\pi\epsilon_0\epsilon_r R}$



* Capacitance depends upon (i) size and shape of conductor (ii) surrounding medium (iii) Presence of other conductor nearby.

Parallel plate Capacitor -

It consist of two metal plates of area A at separation d.



Electric field between the plates

$E = \frac{\sigma}{\epsilon_0}$ $\left[\sigma = \frac{Q}{A}\right]$

Potential difference between the plates $V = E \times d = \frac{\sigma}{\epsilon_0} \times d$

or $V = \frac{Qd}{A\epsilon_0}$

$\therefore C = \frac{Q}{V} \Rightarrow \boxed{C = \frac{\epsilon_0 A}{d}}$

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Force between the plates of a parallel plate capacitor is

$$\boxed{F = \frac{Q^2}{2\epsilon_0 A} = \frac{CV^2}{2d}}$$

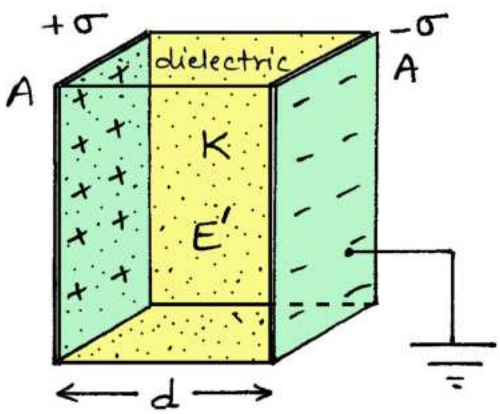
Pressure on plates $\boxed{P = \frac{F}{A} = \frac{Q^2}{2\epsilon_0 A^2} = \frac{\sigma^2}{2\epsilon_0}}$

Effect of dielectric on Capacitance

Case 1. When medium is fully present between the plates -

When dielectric is introduced the electric field between the plates decreases by K (dielectric constant). Due to which V decreases by K.

i.e. $E' = \frac{E}{K}$ and $V' = \frac{V}{K}$

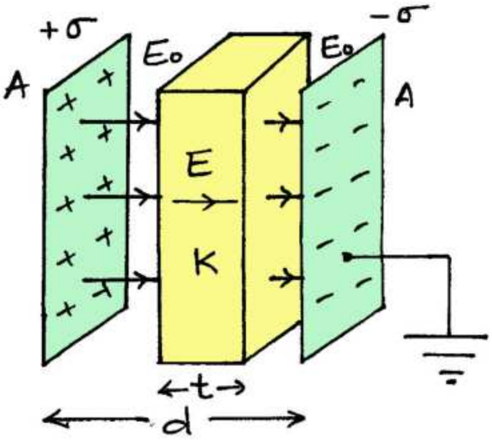


Hence $\boxed{C_{medium} = \frac{K\epsilon_0 A}{d}}$

Case 2. When medium is partially present between the plates

Electric field outside the slab in space $(d-t)$ is $E_0 = \frac{\sigma}{\epsilon_0}$

Electric field inside the slab of thickness t is $E = \frac{\sigma}{K\epsilon_0}$



Net potential difference between the plates

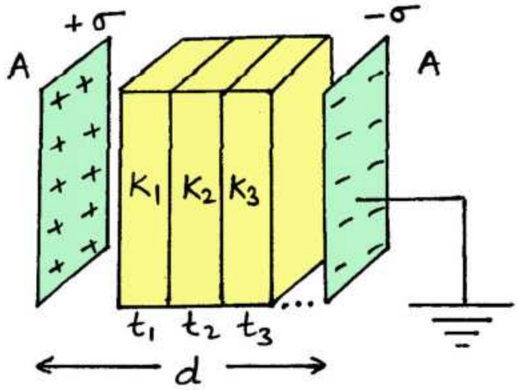
$$V = E_0(d-t) + Et$$

or $V = E_0\left[d - t + \frac{t}{K}\right] \Rightarrow V = \frac{q}{A\epsilon_0}\left[d - t + \frac{t}{K}\right]$

Hence Capacitance $C = \frac{Q}{V} \Rightarrow \boxed{C = \frac{\epsilon_0 A}{\left[d - t + \frac{t}{K}\right]}}$

Case 3. When a number of dielectric slabs are inserted between the plate -

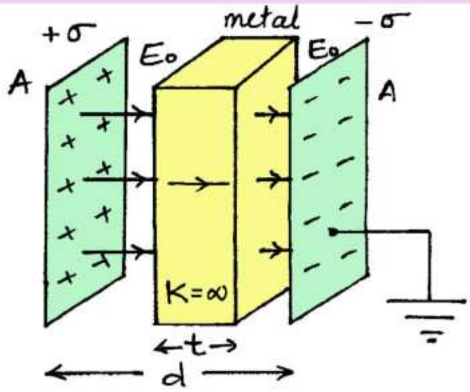
$$\boxed{C = \frac{\epsilon_0 A}{d - (t_1 + t_2 + \ldots) + \left(\frac{t_1}{K_1} + \frac{t_2}{K_2} + \ldots\right)}}$$



Case 4. When a metallic slab is inserted between the plates

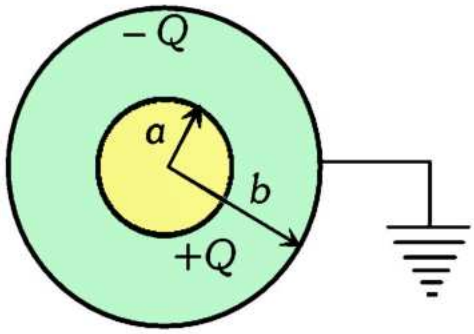
$$C = \frac{\epsilon_0 A}{\left[d - t + \frac{t}{\infty}\right]}$$

$$\boxed{C = \frac{\epsilon_0 A}{(d-t)}}$$



Spherical Capacitor-

It consist of two concentric conducting spheres of radius a and b $(a<b)$. Inner sphere is given charge $+Q$, while outer sphere is earthed.



Potential difference between the spheres is

$$V = \frac{Q}{4\pi\epsilon_0 a} - \frac{Q}{4\pi\epsilon_0 b}$$

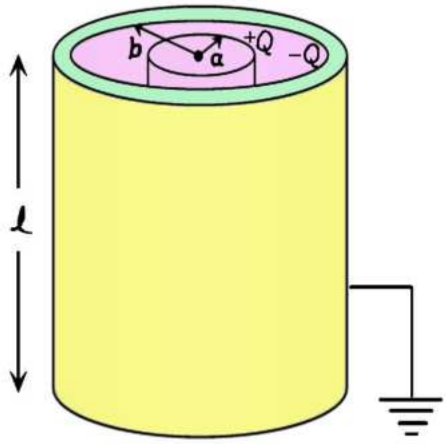
$\therefore$ Capacitance $C = \frac{Q}{V}$ $\Rightarrow$ $\boxed{C = \frac{4\pi\epsilon_0 ab}{(b-a)}}$

When dielectric medium is present between the spheres

$$\boxed{C_{medium} = \frac{4\pi\epsilon_0 K ab}{(b-a)}}$$

Cylinderical Capacitor-

It consist of two co-axial cylinders of radius a and b $(a<b)$, inner cylinder is given charge $+Q$ while outer cylinder is earthed. Common length of cylinders is l.



Electric field between the cylinders at distance r is $E = \frac{2K\lambda}{r} = \frac{2Kq}{rl}$

Potential difference between the cylinders

$$V = \int_a^b \vec{E}\cdot\vec{dr} \Rightarrow V = \frac{2Kq}{l}\int_a^b \frac{1}{r}dr \Rightarrow V = \frac{2Kq}{l}\left[\log r\right]_a^b$$

or $V = \frac{2Kq}{l}\log_e\left(\frac{b}{a}\right) \Rightarrow C = \frac{q}{V} \Rightarrow \boxed{C = \frac{2\pi\epsilon_0 l}{\log_e\left(\frac{b}{a}\right)}}$

Energy stored in a Charged Capacitor -

Let a capacitor of capacitance c is connected to a battery and charges to a potential v.

Let battery supplies small amount of charge dq to the capacitor at constant potential V. Then small amount of work done by the battery is

$$dw = V\,dq \quad \Rightarrow \quad dw = \frac{q}{c}dq$$

Net work done $\quad W = \int_0^Q \frac{q}{c}dq = \frac{1}{c}\left[\frac{q^2}{2}\right]_0^Q$

or $\quad W = \frac{1}{2}\frac{Q^2}{C}$

This work done is stored by the capacitor in the form of the electric potential energy.

Hence $\quad \boxed{U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV}$

Combination of Charged drops -

Suppose we have n identical drops each having radius r, capacitance c, charge q, potential v and energy u.

If these dropes are combined to form a big drop of radius R, capacitance C, charge Q, potential V and energy U then

(i) Charge on big drop $\quad Q = nq$

(ii) Radius of big drop

$$(\text{Volume})_R = n\,(\text{Volume})_r \quad \Rightarrow \quad R = n^{1/3}r$$

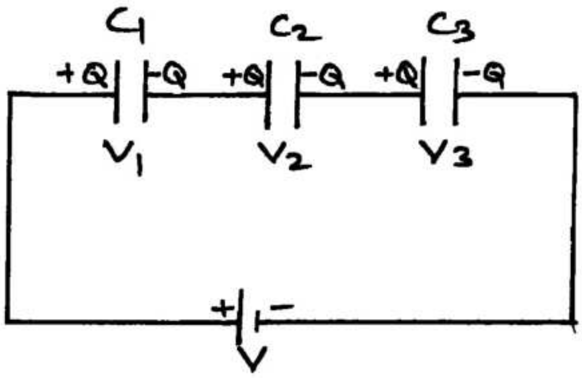
(iii) Capacitance of big drop $\quad C = n^{1/3}c$

(iv) Potential of big drop $\quad V = \frac{Q}{C} \quad \Rightarrow V = n^{2/3}v$

(v) Energy of big drop $\quad U = \frac{1}{2}CV^2 \quad \Rightarrow U = n^{5/3}u$

Combination of Capacitor

1.) Capacitors in series -



In series combination the charge on each capacitor is equal.

i.e. $Q = C_1V_1 = C_2V_2 = C_3V_3$

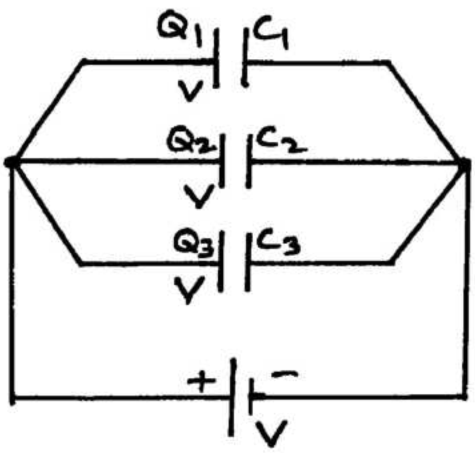
The total potential difference is shared by capacitors in the inverse ratio of the capacitance.

$$V = V_1 + V_2 + V_3$$

If C_s is the net capacitance, then

$$\frac{Q}{C_s} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \Rightarrow \boxed{\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

2.) Capacitors in parallel -



In parallel combination the potential difference across each capacitor is same and equal to the total potential applied.

i.e $V = V_1 = V_2 = V_3$

The total charge is shared by each capacitor in the direct ratio of the capacitances.

$$Q = Q_1 + Q_2 + Q_3$$

If C_P is the net capacitance then,

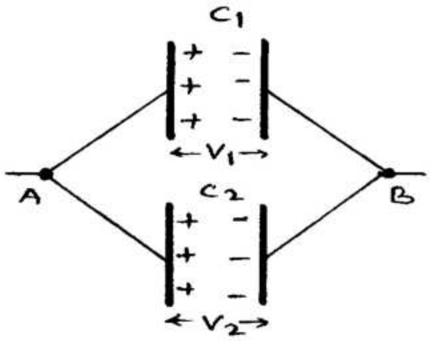
$$C_PV = C_1V + C_2V + C_3V \Rightarrow \boxed{C_P = C_1 + C_2 + C_3}$$

Note- (i) If N identical capacitors are connected then
$C_{series} = \frac{C}{N}$ and $C_{parallel} = NC$

(ii) For a given voltage to store maximum energy capacitors should be connected in parallel.

Common Potential

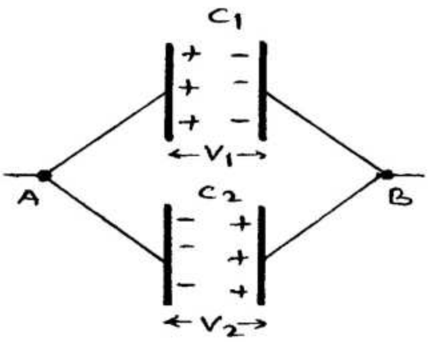
(i)



$$\text{Common Potential} = \frac{\text{Total charge}}{\text{Total Capacity}}$$

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

(ii)

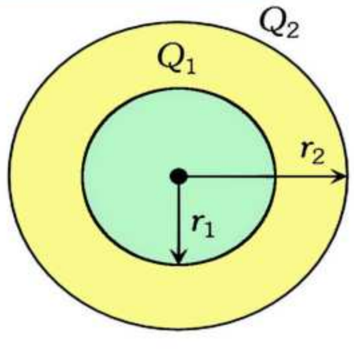


$$\text{Common Potential} = \frac{\text{Total charge}}{\text{Total Capacity}}$$

$$V = \frac{C_1V_1 - C_2V_2}{C_1 + C_2}$$

Potential Due to Concentric Spheres -

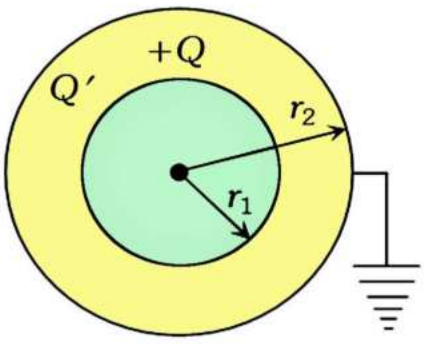
1.| Potential at the surface of each shell



$$V_1 = \frac{KQ_1}{r_1} + \frac{KQ_2}{r_2}$$

$$V_2 = \frac{KQ_1}{r_2} + \frac{KQ_2}{r_2}$$

2.| Potential at the surface of outer sphere



$$V_2 = \frac{KQ}{r_2} + \frac{KQ'}{r_2} = 0$$

$$\Rightarrow \quad Q' = -Q$$

Potential of the inner sphere

$$V_1 = \frac{KQ}{r_1} + \frac{K(-Q)}{r_2}$$

$$V_1 = \frac{KQ}{r_1} - \frac{KQ}{r_2}$$