Newton's Law of Motion

Point Mass -

An object can be considered as a point mass if during motion in a given time, it coveres distance much greater than its own size.

Inertia -

It is the property of the body due to which the body opposes the change of itself state.
Inertia of a body is measured by mass of the body.

Inertia ∝ mass

Hevier the body, greater is the force required to change its state and hence greater is the inertia.

* Inertia has not unit and no dimensions.
* Two body of equal mass, one in motion and another is at rest, has the same inertia because it is a factor of mass only and does not depend upon the velocity.

Types of inertia

1.] Inertia of rest -

It is the inability of a body to change by itself, its state of rest. This means a body at rest remains at rest and cannot start moving by its own.

Eg. A person who is standing freely in bus, thrown backward, when bus starts suddenly.

2.] Inertia of Motion-

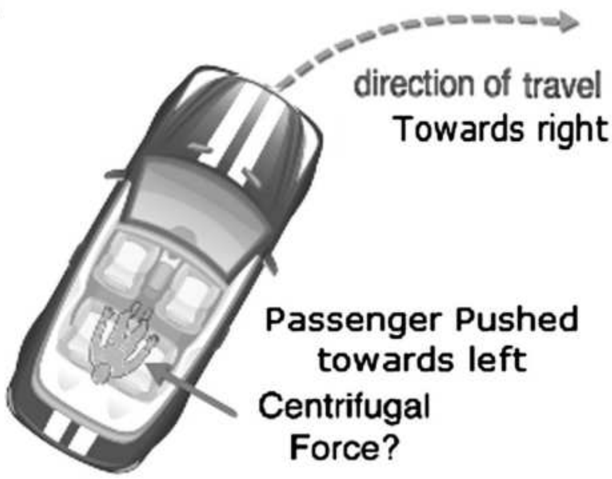
It is the in ability of the body to chang by itself its state of uniform motion.

Eg. A person jumps out of a moving train may fall forward.

3.] Inertia of direction-

It is the inability of a body to change by itself its direction of motion.

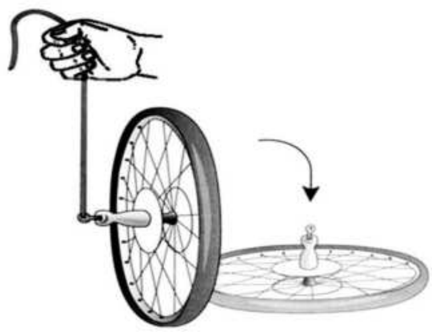
Eg. When a car goes round a curve suddenly, the person sitting inside it, is thrown outwards.



4.] Inertia of Rotation-

It is inability of a body to change by itself its rotational axis.

Eg. It Keeps balance of moving cycle.

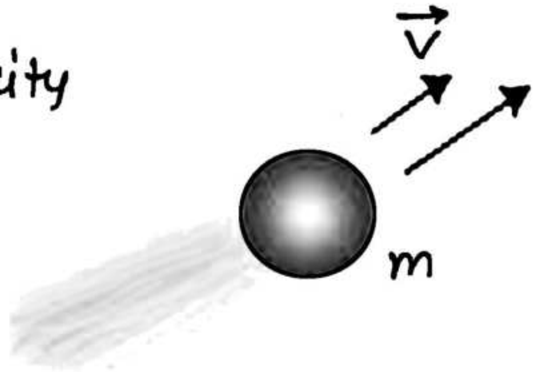


Linear Momentum –

The total quantity of motion possessed by a moving body is known as the momentum of the body. It is measured as the product of the mass of the body and its velocity.

Momentum = mass x velocity

$$\vec{p} = m\vec{v}$$

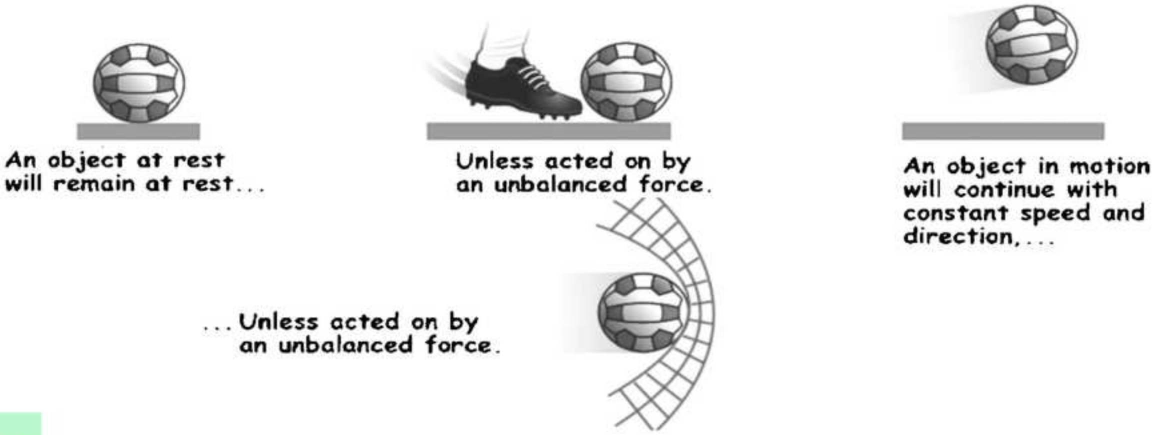


It is a vector quantity and its direction is same as the direction of velocity of the body.

SI Unit – Kg.m/sec. Dimensions – $[M^1L^1T^{-1}]$

Newton's First Law -

Everybody continues in its state of rest or uniform motion along a straight line, unless it is acted upon by some external force to change the state.



Force -

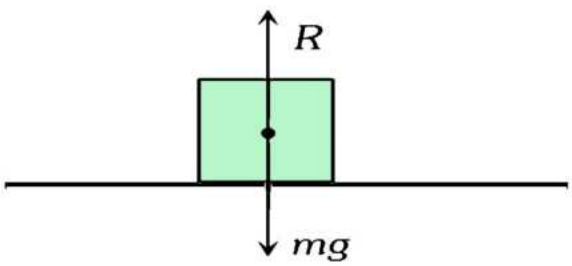
Force is that push or pull which changes or tends to change the state of rest or of uniform motion in a straight line.

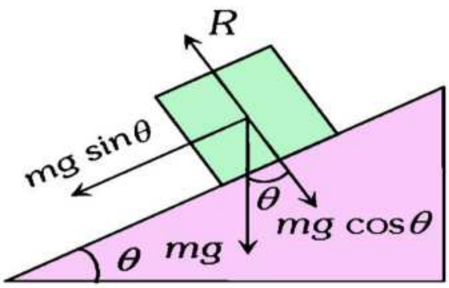
SI Unit - Newton Dimensions - $[M^1 L^1 T^{-2}]$

Common Forces in Mechanics

1.] Weight - Weight of a body is the force with which earth attracts it. Its magnitude is given by mg. It is also known as gravitational force.

2.] Reaction or Normal Force - When a body is placed on a rigid surface, then the body experiences a force which is perpendicular to the surface in contact. This force is called reaction or normal force.

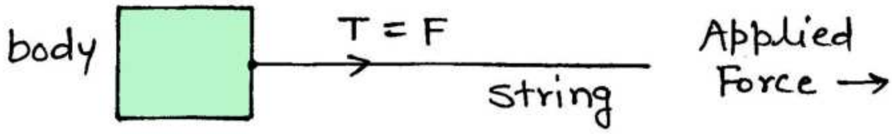




3.) Tension -

The force exerted by the end of taut string, rope or chain against pulling (applied) force is called the tension.

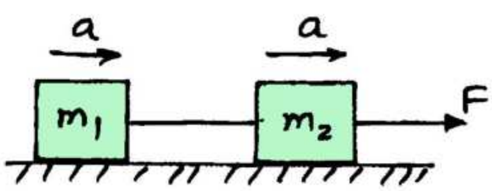
The direction of tension is away from the body.

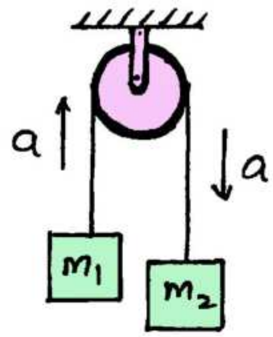


IMPORTANT POINTS -

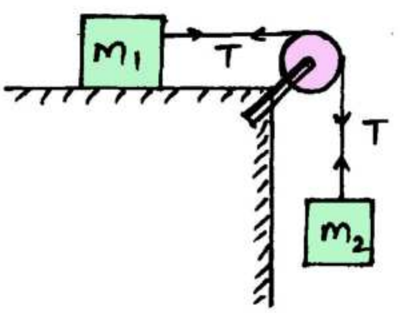
(i) If string is inextensible then the magnitude of acceleration of any number of masses connected through string is always same.

(ii) If string is massless then the tension in it is same everywhere.





(iii) If there is no friction between pulley and string then the tension will be same on both sides of pulley.



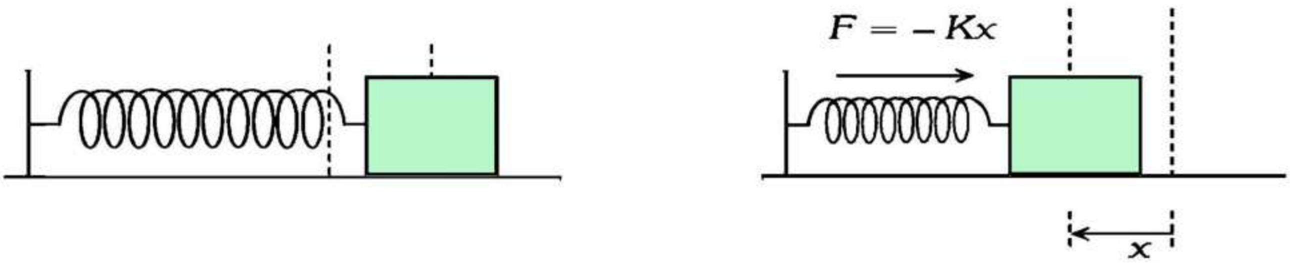
NOTE -
If there is friction between string and pulley then, tension will be different on both sides of pulley.

4.] Spring Force -

Every spring resists any change in its length. This resistive force increases with change in length. It is given by -

$$F = -Kx$$

Here x is the change in length and K is the spring constant.



Newton's Second Law -

The rate of change of linear momentum of a body is directly proportional to the external force applied on it and this change in momentum takes place in the direction of the applied force.

Hence $F \propto \frac{d\vec{p}}{dt}$ or $\vec{F} = \frac{d\vec{p}}{dt}$

$\Rightarrow F = m\frac{d\vec{v}}{dt}$ $\Rightarrow$ $\vec{F} = m\vec{a}$

* Newton's second law gives the quantitative definition of force.

APPLICATIONS OF NEWTON'S SECOND LAW OF MOTION

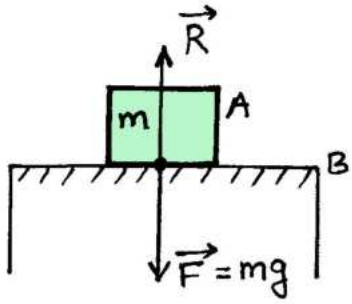
➢ A cricketer lowers his hands while catching the ball.

$$F = \frac{m(v-u)}{t}$$

Newton's Third Law-

To every action, there is always an equal and opposite reaction.

If a body A exerts force $\vec{F}$ on another body B, then B exerts a reaction force $\vec{R} = -\vec{F}$ on A. The two forces are acting along the same line.



Note- Action and reaction force acts on different bodies hence they never cancel each other.

* Newton's third law gives the nature of the force (i.e. forces always occur in pairs.)

When you push on the wall, the wall pushes on you.

Impulse-

When a large force act for an extremely short duration, neither the magnitude of the force nor the time for which it acts is important. In such a case, the total effect of force is measured.

The total effect of force is called **impulse**.

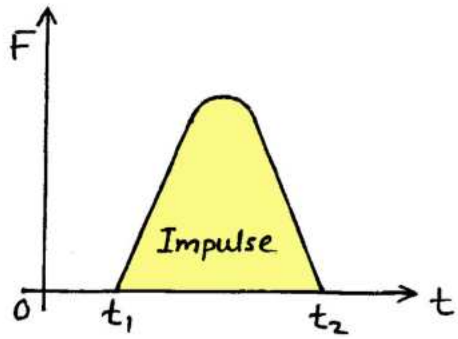
If a large force acts on a body for a small time then

Impulse = Force × time

$$\text{Impulse} = \vec{F} \times dt$$

For a finite interval of time from t_1 to t_2 is

$$\text{Impulse} = \int_{t_1}^{t_2} \vec{F} dt$$



Note -

(i) For Force - Time graph impulse is equal to the area under the curve.

(ii) From Newton's second Law $\vec{F} = \frac{d\vec{P}}{dt}$

or $\int_{t_1}^{t_2} \vec{F} dt = \int_{P_1}^{P_2} d\vec{P}$

or Impulse $\vec{I} = \vec{P_2} - \vec{P_1} = \Delta \vec{P}$

Hence impulse of a force is equal to the change in momentum. It is known as 'Impulse momentum theorem'.

(iii) If F_{av} is the average magnitude of the force then

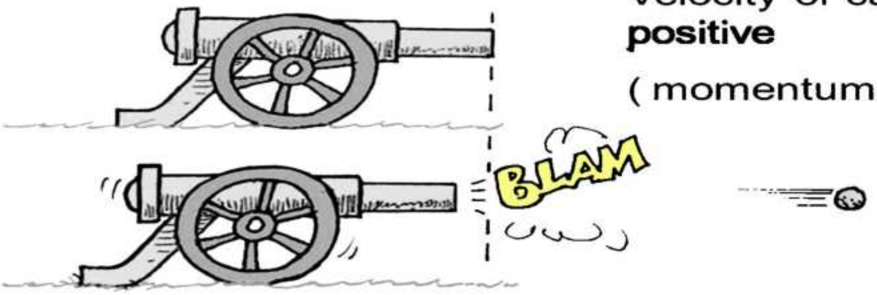
$$I = \int_{t_1}^{t_2} F_{av} dt = F_{av} \int_{t_1}^{t_2} dt = F_{av} \cdot \Delta t$$

Law of Conservation of Linear momentum -

If no external force acts on a system (Isolated System) of constant mass, the total momentum of the system remains constant with time.

If $F_{ext} = 0$ then $P_{initial} = P_{final}$

The momentum before firing is zero. After firing, the net momentum is still zero because the momentum of the cannon is equal and opposite to the momentum of the cannonball.



Velocity cannon to left is **negative**

Velocity of cannonball to right is **positive**

(momentums cancel each other out)

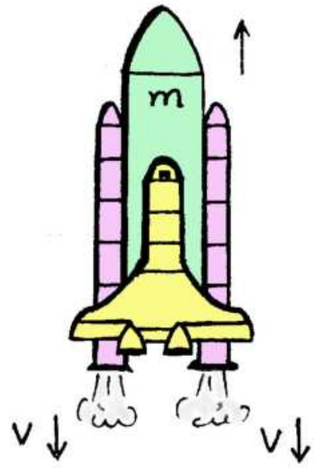
Free body diagram -

It is a diagram of the object isolated from its surroundings, with forces applied to the body by the various other bodies that interact with it.

Note - Forces that a body exerts on itself or on other body are never included in free body diagram.

Rocket Propulsion –

The initial momentum of the rocket on its launching pad is zero. When it is fired, the exhaust gases rush downward at a high speed and to conserve momentum the rocket moves upwards.



Thrust on the rocket $F = -\frac{V\,dm}{dt}$

Here negative sign indicates that direction of thrust is opposite to the direction of escaping gases.

Acceleration of the rocket $a = \frac{V}{m}\frac{dm}{dt}$

V = Velocity of exhaust gases

$\frac{dm}{dt}$ = rate of fuel consumption = rate of ejection of the fuel

Motion of a body in Lift

When a body of mass m is placed on a weighing machine which is placed in a lift, then actual weight of the body is mg.

This force mg acts on a weighing machine which offers a reaction R given by the reading of weighing machine.

Note - This reaction R exerted by the surface of contact on the body is the apparent weight of the body.

1.] When the lift is moving upwards -

Net upward force on lift is

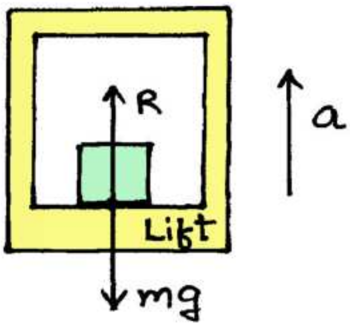
$$F = R - mg$$

or $$ma = R - mg$$

Hence apparent weight (R) is

$$R = m(g+a)$$

Apparent Weight > Actual Weight



2.] When the lift is moving downward -

Net downward force on Lift is

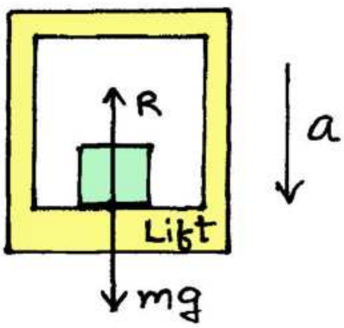
$$F = mg - R$$

or $$ma = mg - R$$

Hence apparent weight (R) is

$$R = m(g-a)$$

Apparent Weight < Actual Weight



Note - Here $m = m_1 + m_2$ [Mass of lift + Mass of body]

Note -

(i) If lift is at rest or moving upward or downward with constant velocity then $a = 0$ and $R = mg$.

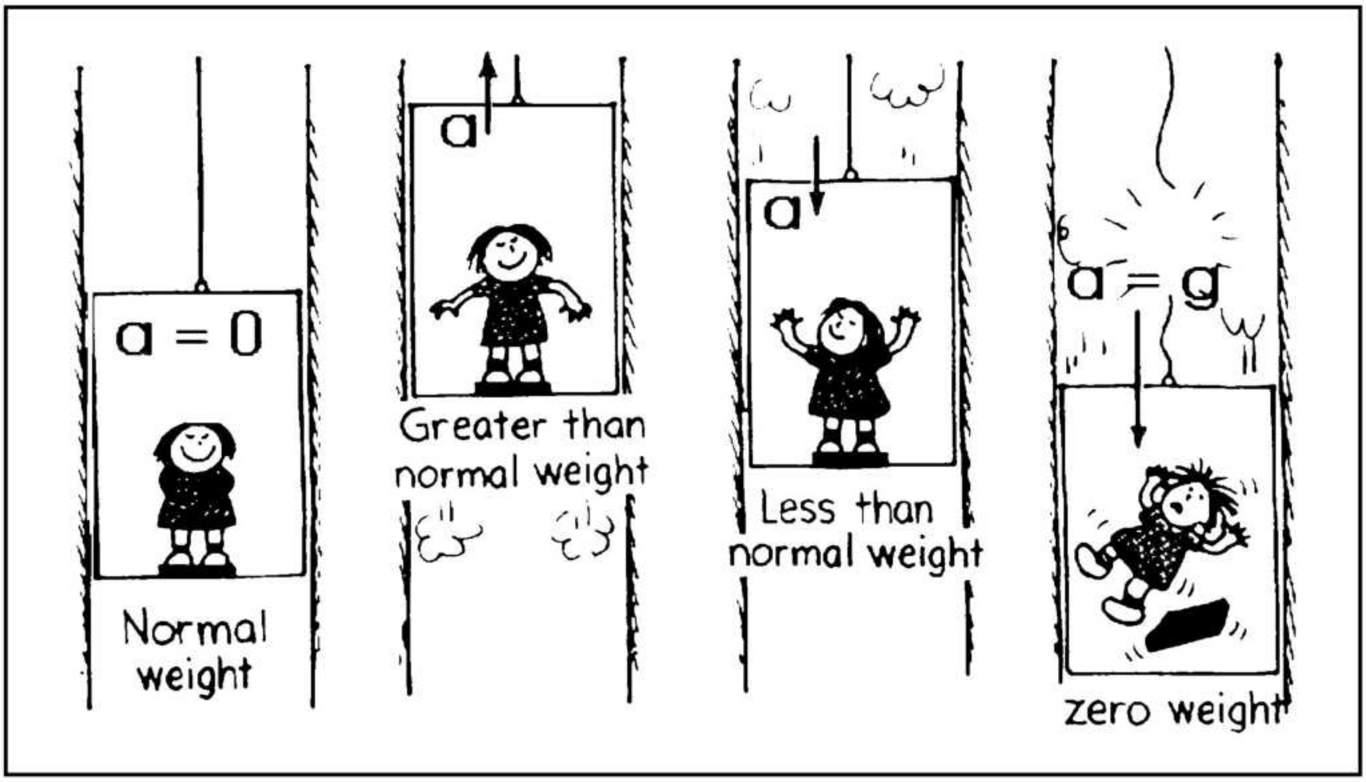
Hence **Apparent Weight = Actual Weight**

(ii) If lift is accelerating downword at the rate of 'g' then $R = 0$.

Hence **Apparent Weight = 0 (Weightlessness)**

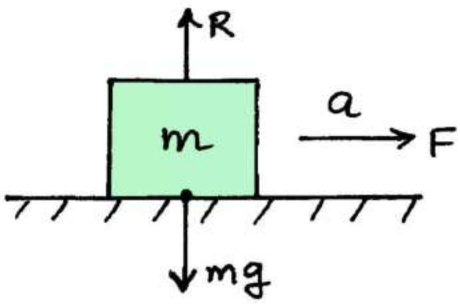
(iii) If lift is accelerating downword at the rate of $a > g$ then $R = -ve$.
Hence Apparent weight is $-ve$, which means the body will rise from the floor of the lift and stick to the ceiling of the lift.

> **Note-**
> अगर Lift पर retardation work कर रहा है तो acceleration 'a' की direction हमें lift की velocity के opposite लेनी होती है।



Acceleration of Block on Horizontal smooth surface.

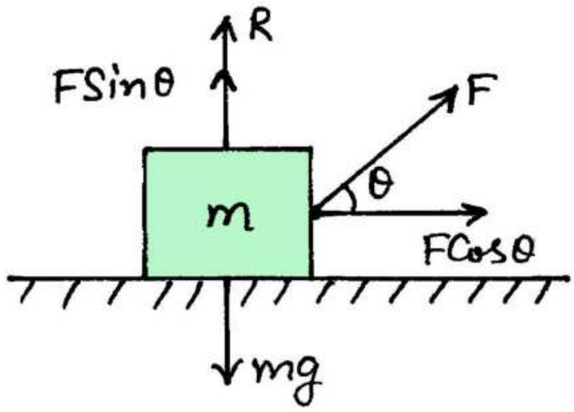
(i) When a pull is horizontal -



Here $R = mg$ and $F = ma$

$\therefore$ $a = \frac{F}{m}$

(ii) When a pull is acting at an angle θ to the horizontal -



$R + F\sin\theta = mg$

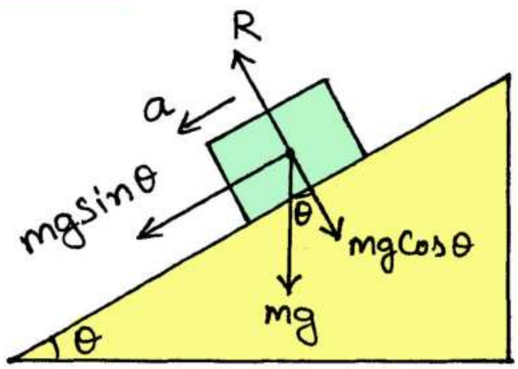
or $R = mg - F\sin\theta$

and $F\cos\theta = ma$

$\therefore$ $a = \frac{F\cos\theta}{m}$

Acceleration of Block on smooth incined Plane.

(i) When inclined plane is at Rest -



Normal Reaction $R = mg\cos\theta$

Force on object along the incined plane

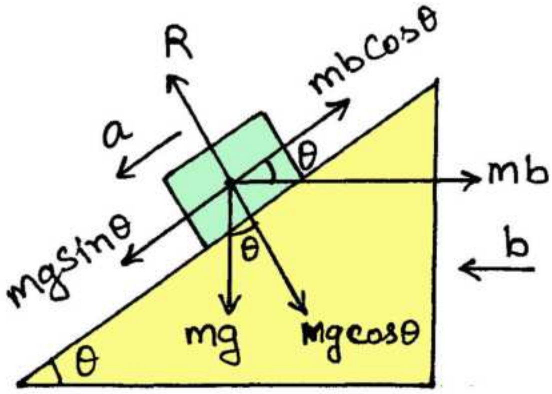
$F = mg\sin\theta$

or $ma = mg\sin\theta$

$\therefore$ $a = g\sin\theta$

(ii) When inclined plane is given a horizontal acceleration 'b' -

Since the body lies in an accelerating frame, an initial force (mb) acts on it in the opposite direction.



Normal Reaction

$$R = mg\cos\theta + mb\sin\theta$$

and $F = ma = mg\sin\theta - mb\cos\theta$

$$\therefore \quad a = g\sin\theta - b\cos\theta$$

Note - The condition for the body to be at rest relative to the inclined plane is that $a = 0$.

Hence $a = g\sin\theta - b\cos\theta = 0$

$$\therefore \quad b = g\tan\theta$$

Inertial Frame of reference -

A non-accelerating frame of reference is called an inertial frame of reference. A frame of reference moving with a constant velocity is an inertial frame of reference.

Non-inertial Frame of reference -

An accelerating frame of reference is called a non-inertial frame of reference.

Note- A rotating frame of reference is a non-inertial frame of reference because it is also an accelerating one (Centripetal acceleration).

Pseudo Force (force due to inertia) -

Pseudo force is a physically apparent but non-existent force that felt by an observer in a noninertial frame (a frame undergoing acceleration)

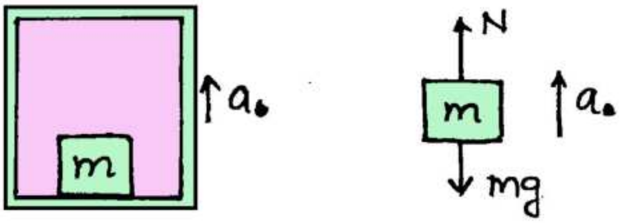
* Pseudo force is not a real force. When we draw a free body diagram of a mass, with respect to an inertial frame of reference, then we apply only the real forces. (Forces which are actually acting on the mass)

* But when a free body diagram is drawn for a non-inertial frame of reference a "Pseudo force" (in addition to all real forces) has to be applied to make the equation $F = ma$ valid in this frame also.

Note- In case of rotating frame of reference this pseudo force is called the centrifugal force when applied for centripetal acceleration.

Examples-

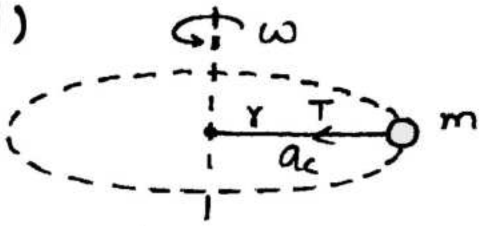
(i)



$N - mg = ma$

↑ Pseudo force

(ii)

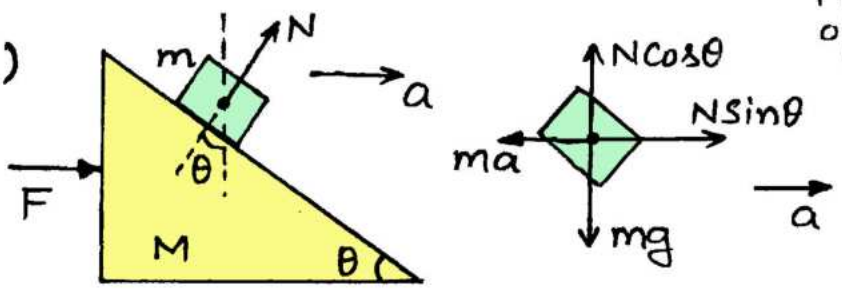


Centripetal acceleration = $r\omega^2$

$T = mr\omega^2$ (centrifugal force)

↑ Pseudo force

(iii)

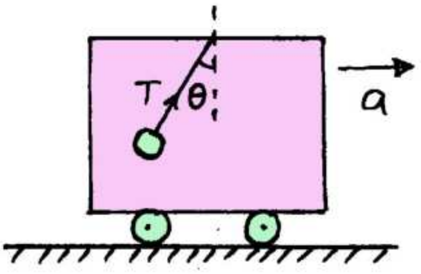


From non-inertial frame of reference (Wedge)

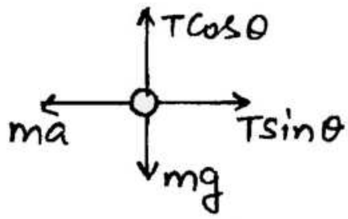
$N\sin\theta = ma$

↑ Pseudo force

(iv)



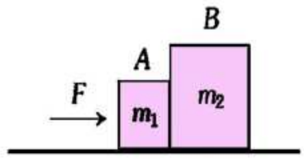
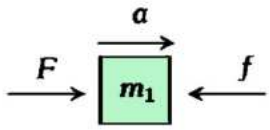
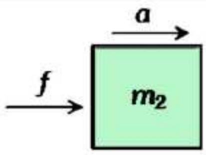
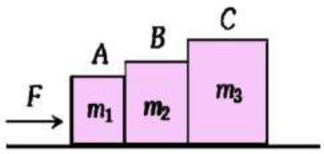
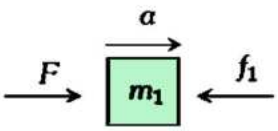
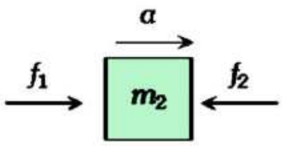
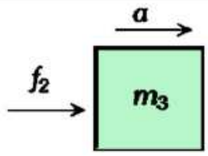
From non-inertial frame of reference (Car)



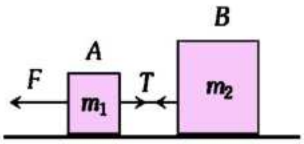
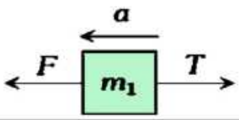
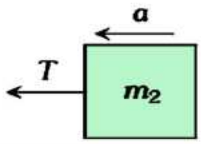
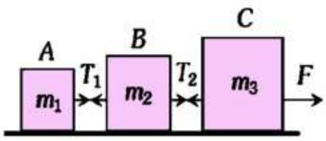
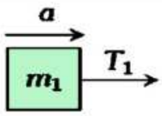
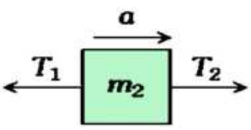
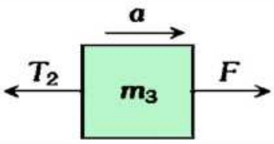
$T\sin\theta = ma$

↑ Pseudo force

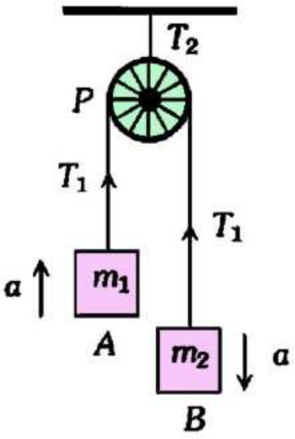
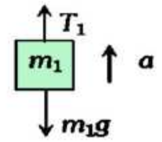
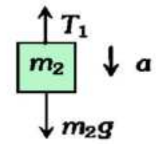
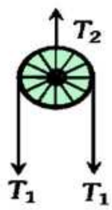
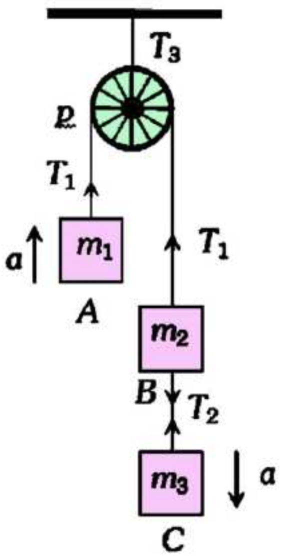
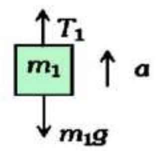
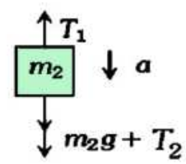
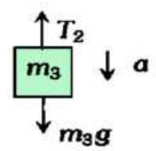
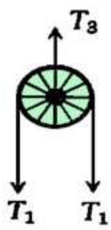
Motion of Blocks in Contact

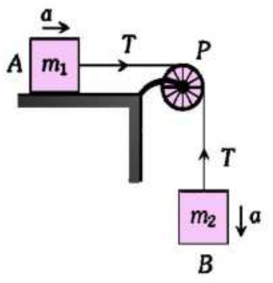
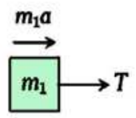
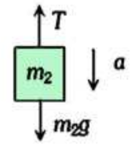
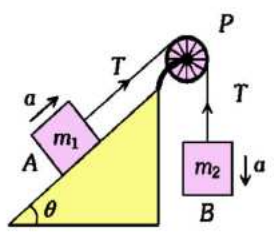
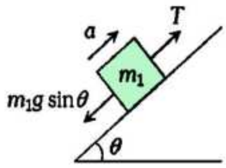
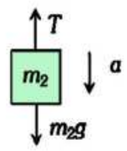
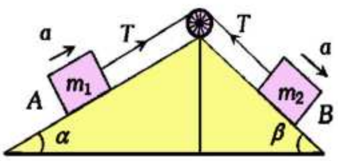
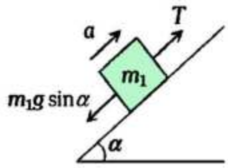
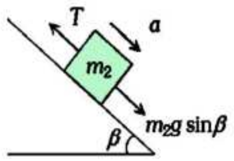
Condition	Free body diagram	Equation	Force and acceleration
		$F - f = m_1 a$	$a = \dfrac{F}{m_1 + m_2}$
		$f = m_2 a$	$f = \dfrac{m_2 F}{m_1 + m_2}$
		$F - f_1 = m_1 a$	$a = \dfrac{F}{m_1 + m_2 + m_3}$
		$f_1 - f_2 = m_2 a$	$f_1 = \dfrac{(m_2 + m_3)F}{m_1 + m_2 + m_3}$
		$f_2 = m_3 a$	$f_2 = \dfrac{m_3 F}{m_1 + m_2 + m_3}$

Motion of Blocks connected by Mass less string

Condition	Free body diagram	Equation	Tension and acceleration
		$F - T = m_1 a$	$a = \dfrac{F}{m_1 + m_2}$
		$T = m_2 a$	$T = \dfrac{m_2 F}{m_1 + m_2}$
		$T_1 = m_1 a$	$a = \dfrac{F}{m_1 + m_2 + m_3}$
		$T_2 - T_1 = m_2 a$	$T_1 = \dfrac{m_1 F}{m_1 + m_2 + m_3}$
		$F - T_2 = m_3 a$	$T_2 = \dfrac{(m_1 + m_2)F}{m_1 + m_2 + m_3}$

Motion of Connected Block over a pulley

Condition	Free body diagram	Equation	Tension and acceleration
		$m_1a = T_1 - m_1g$	$T_1 = \frac{2m_1m_2}{m_1+m_2}g$
		$m_2a = m_2g - T_1$	$T_2 = \frac{4m_1m_2}{m_1+m_2}g$
		$T_2 = 2T_1$	$a = \left[\frac{m_2-m_1}{m_1+m_2}\right]g$
		$m_1a = T_1 - m_1g$	$T_1 = \frac{2m_1[m_2+m_3]}{m_1+m_2+m_3}g$
		$m_2a = m_2g + T_2 - T_1$	$T_2 = \frac{2m_1m_3}{m_1+m_2+m_3}g$
		$m_3a = m_3g - T_2$	$T_3 = \frac{4m_1[m_2+m_3]}{m_1+m_2+m_3}g$
		$T_3 = 2T_1$	$a = \frac{[(m_2+m_3)-m_1]g}{m_1+m_2+m_3}$

Condition	Free body diagram	Equation	Tension and acceleration
		$T = m_1 a$	$a = \frac{m_2}{m_1 + m_2} g$
		$m_2 a = m_2 g - T$	$T = \frac{m_1 m_2}{m_1 + m_2} g$
		$m_1 a = T - m_1 g \sin\theta$	$a = \left[\frac{m_2 - m_1 \sin\theta}{m_1 + m_2}\right] g$
		$m_2 a = m_2 g - T$	$T = \frac{m_1 m_2 (1 + \sin\theta)}{m_1 + m_2} g$
		$T - m_1 g \sin\alpha = m_1 a$	$a = \frac{(m_2 \sin\beta - m_1 \sin\alpha)}{m_1 + m_2} g$
		$m_2 a = m_2 g \sin\beta - T$	$T = \frac{m_1 m_2 (\sin\alpha + \sin\beta)}{m_1 + m_2} g$

Pulleys -

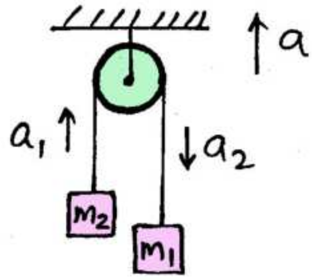
The problems based on pulleys becomes very easy using pulling force method.

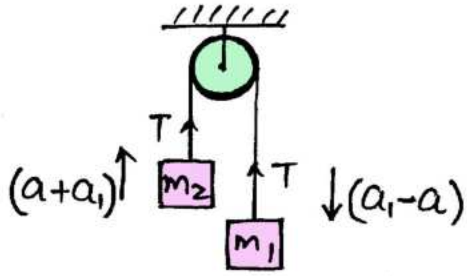
$$\text{Acceleration of the system} = \frac{\text{Net Pulling force}}{\text{Total mass to be pulled}}$$

Note-

इस type के Questions में पहले acceleration of blocks relative to pulley 'a_1', calculate कर लेंगे। इसके बाद System का acceleration उसमें direction के according add या subtract कर देंगे।

इसके बाद FBD से equations बना कर easily question को solve कर सकते हैं।

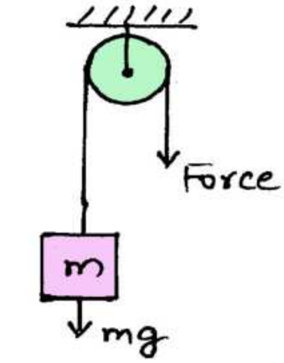




Mechanical Advantage

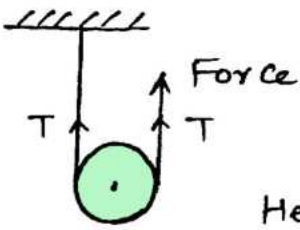
The ratio of the force produced by a machine to the force applied to it, is called mechanical advantage of the machine. It is used in assessing the performance of a machine.

$$\text{Mechanical Advantage} = \frac{\text{Output force}}{\text{Input force}} = \frac{F_{out}}{F_{in}}$$



$F_{out} = F_{in}$

$M.A. = 1$



$2T = mg$

$T = \frac{mg}{2}$

Here $F_{in} = T$

$M.A. = \frac{F_{out}}{F_{in}} = \frac{mg}{mg/2}$

$M.A. = 2$