

1. Electric charges & field

Electric Charge It is the property associated with matter due to which it produces and experiences electrical and magnetic effects.

The excess or deficiency of electrons in a body gives the concept of charge.

SI Unit - Coulomb (C) or Ampere x Sec. (A.s)
Dimensions - [AT]

Basic Properties of Charge

- 1.] Charge is a scalar quantity.
- 2.] Charge is always associated with mass.
 - * When body gets +ve charge its mass decreases.
 - * When body gets -ve charge its mass increases.
- 3.] Charge is quantised. $Q = \pm ne$
where n = any integer value and $e = 1.6 \times 10^{-19} \text{ C}$
- 4.] Charge is conserved. (But mass is not conserved)
- 5.] Similar Charges repel and dissimilar charges attract each other.



- 6.] Accelerated charge radiates energy.

$V = 0$
 $Q (+)$

Produces only $\vec{E}$

$V = \text{Const.}$
 $Q (+)$

Produces only $\vec{E}$ & $\vec{B}$

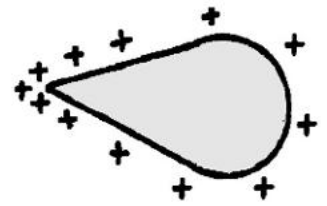
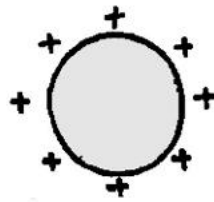
$V \neq \text{Const.}$
 $Q (+)$

Produces $\vec{E}$ & $\vec{B}$ and radiates energy

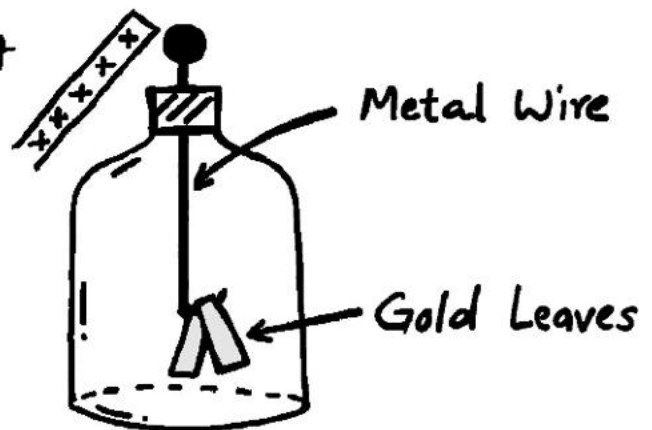
- 7.] True test of electrification is repulsion, not attraction.
- 8.] Charge given to a conductor always resides on its surface. In insulator it remains where it is placed i.e. may reside inside the body also.
- 9.] Charge density of a irregular conducting body is not uniform. It is maximum where the radius of curvature is minimum and vice-versa. i.e. $\sigma \propto \frac{1}{R}$.
This is why charge leaks from sharp points.

Charge density

$$\sigma \propto \frac{1}{R}$$



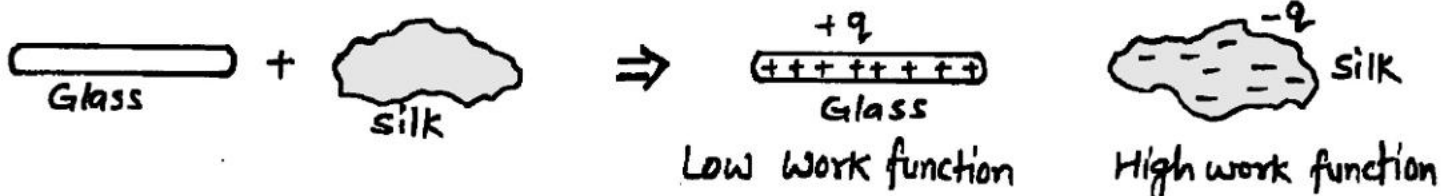
- 10.] The apparatus used to detect charge on a body is "Gold Leaf Electroscope".



Methods of Charging

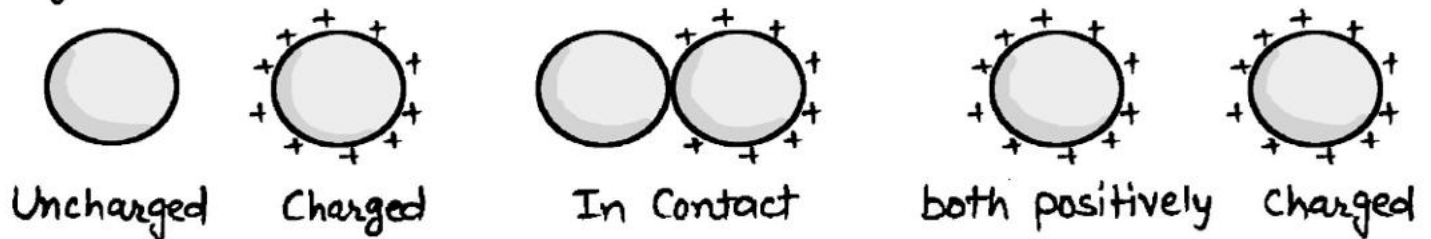
1.] Friction

By rubbing two bodies together both positive and negative charges in equal amount appears due to transfer of electron from lower work function body to body of higher work function.

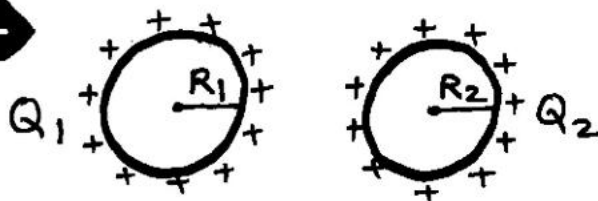


2.] Conduction

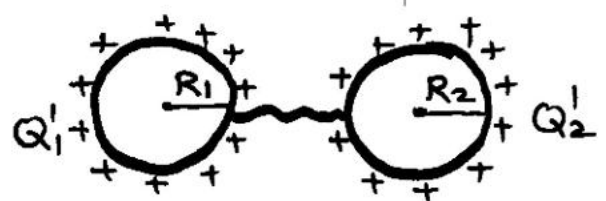
The process of transfer of charge by contact of two bodies is known as conduction. Positive charge flows from higher potential to lower potential, while negative charge flows from lower to higher potential.



Note-



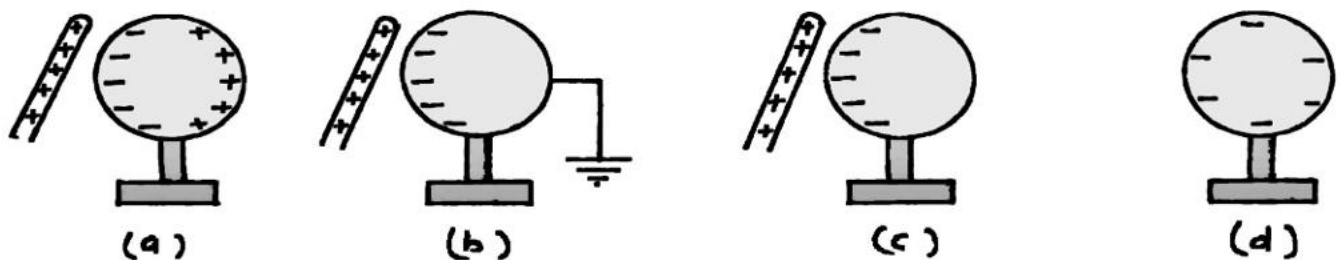
$$Q'_1 = \frac{R_1}{R_1 + R_2} (Q_1 + Q_2)$$



$$Q'_2 = \frac{R_2}{R_1 + R_2} (Q_1 + Q_2)$$

3.] Induction

If a charged body is brought near a neutral body, the charged body will attract opposite charge and repel similar charge present in the neutral body. As a result one side of the neutral body becomes positive and the other negative, this process is known induction.



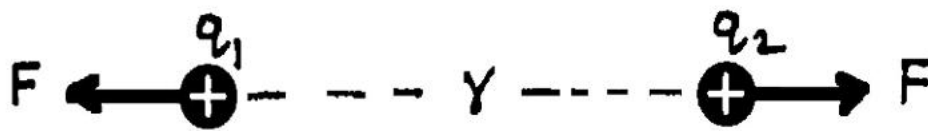
Induced charge can be lesser or equal to inducing charge (but never greater) and its maximum value is given by

$$Q' = -Q \left[1 - \frac{1}{K} \right] \quad K \rightarrow \text{dielectric Const.}$$

Coulomb's Law

The force of interaction between any two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them, and acts along the line joining the two charges.

Suppose two point charges q_1 and q_2 are separated in vacuum by a distance r .



According to Coulomb's Law,

$$F \propto q_1 q_2$$

and $F \propto \frac{1}{r^2}$

or

$$F = \frac{K q_1 q_2}{r^2}$$



Charles Augustin de Coulomb

Note -

* Electrostatic constant K depends on the nature of medium separating the charges and on the system of units.

In SI system, $K = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2/\text{C}^2$

* ϵ_0 electrical permittivity of free space

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2$$

Coulomb's Law in vector form

Let the position vector of charges q_1 and q_2 be $\vec{r}_1$ and $\vec{r}_2$ respectively.

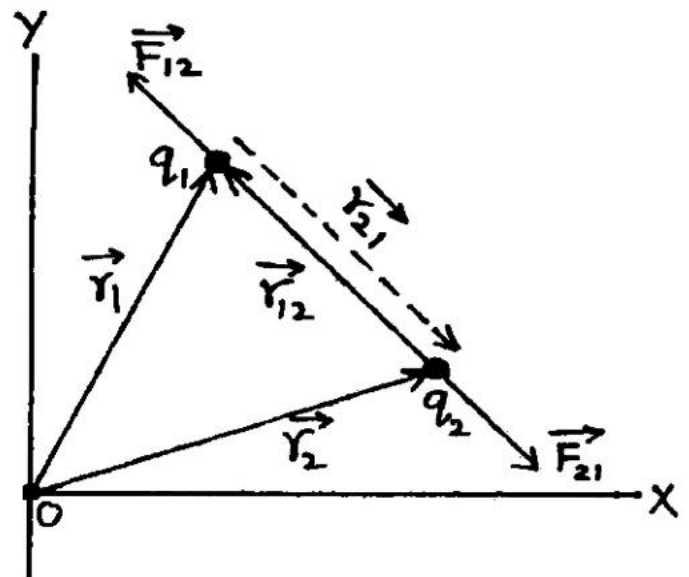
$$|\vec{F}_{12}| = |\vec{F}_{21}|$$

Here $\vec{r}_{21} = \vec{r}_2 - \vec{r}_1$

also $\vec{r}_{12} = \vec{r}_1 - \vec{r}_2$

and $\vec{r}_{12} = -\vec{r}_{21}$

$$\hat{r}_{12} = \frac{\vec{r}_{12}}{|\vec{r}_{12}|} \quad \text{and} \quad \hat{r}_{21} = \frac{\vec{r}_{21}}{|\vec{r}_{21}|}$$



$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{|\vec{r}_{12}|^2} \hat{r}_{12}$$

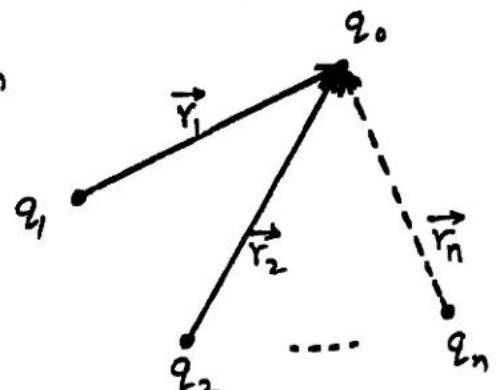
$$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{|\vec{r}_{21}|^2} \hat{r}_{21}$$

Force between multiple charges

According to the principle of superposition, total force acting on a given charge due to number of charges is the vector sum of the individual forces acting on that charge due to all the charges.

$$\vec{F} = \frac{Kq_0q_1}{r_1^2} \hat{r}_1 + \frac{Kq_0q_2}{r_2^2} \hat{r}_2 + \dots + \frac{Kq_0q_n}{r_n^2} \hat{r}_n$$

or $\vec{F} = Kq_0 \sum_{i=1}^n \frac{q_i}{r_i^2} \hat{r}_i$



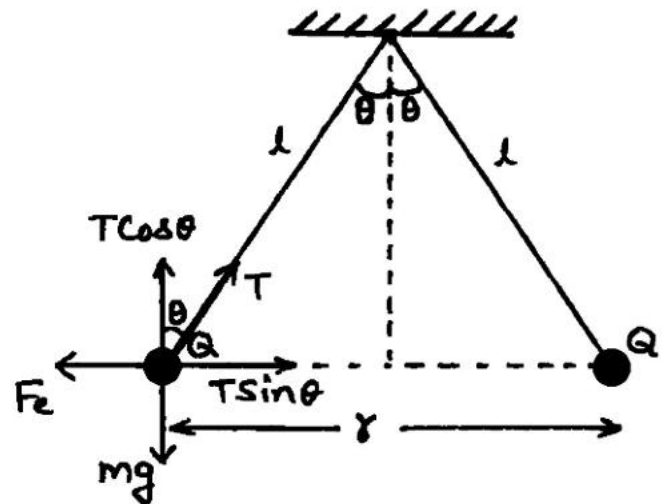
Equilibrium of suspended point charge system

For equilibrium position

$$T \cos \theta = mg$$

$$T \sin \theta = F_e = \frac{KQ^2}{r^2}$$

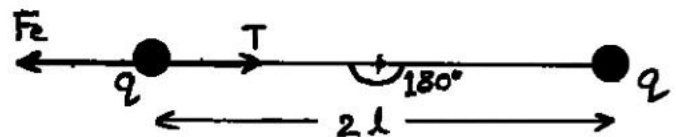
$$\text{or } \tan \theta = \frac{F_e}{mg} = \frac{KQ^2}{r^2 mg}$$



Note-

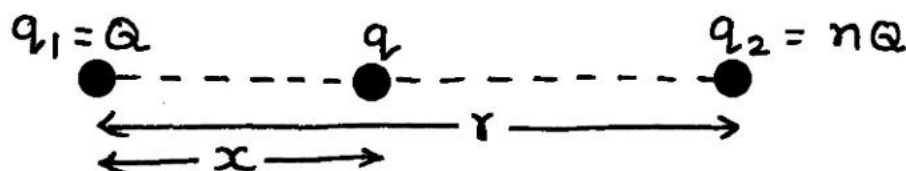
If whole setup is taken into an satellite (where $g \approx 0$) then

$$T = F_e = \frac{Kq^2}{4l^2}$$



Neutral Point

Position of 3rd charge where force on that charge will be zero.



For Like charges

$$x = \frac{r}{\sqrt{n} + 1}$$

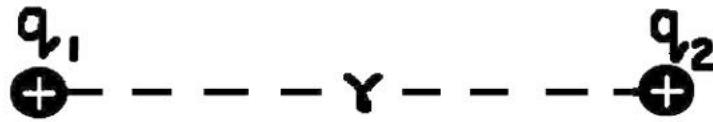
For Unlike Charges

$$x = \frac{r}{\sqrt{n} - 1}$$

Where $n = \frac{\text{Large charge}}{\text{Small charge}}$

Dielectric Constant (K)

Dielectric constant of a medium is the ratio of absolute electrical permittivity of the medium to the electrical permittivity of free space.



$$\text{Force in vacuum } (F_0) = \frac{1}{4\pi\epsilon_0} \times \frac{q_1 q_2}{r^2} \quad \text{--- (1)}$$

$$\text{Force in a medium } (F_m) = \frac{1}{4\pi\epsilon} \times \frac{q_1 q_2}{r^2} \quad \text{--- (2)}$$

where ϵ = electrical permittivity of the medium.

Dividing eq. (1) by (2), we get-

$$\frac{F_0}{F_m} = \frac{\epsilon}{\epsilon_0} = \epsilon_r \text{ or } K$$

where ϵ_r or K is called dielectric constant or relative electrical permittivity of the medium.

Note -

* Thus force between two given charges in a medium (K) is only $\frac{1}{K}$ times of the force between them in air/vacuum.

$$F_m = \frac{F_0}{K}$$

* For All Metals $\epsilon_r = \infty$

* For Air $\epsilon_r = 1$

$$1 \leq \epsilon_r \leq \infty$$

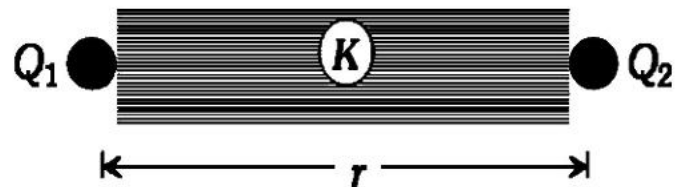


Effect of medium on Coulomb's Force

- (a) When a dielectric medium is completely filled in between charges, rearrangement of charges inside the dielectric medium take place and the force between the charges decreases by a factor of K (dielectric Const.)

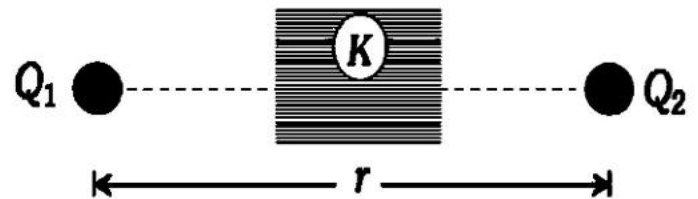
$$F_{\text{medium}} = \frac{F_{\text{air}}}{K}$$

$$F_{\text{medium}} = \frac{1}{4\pi\epsilon_0 K} \cdot \frac{Q_1 Q_2}{r^2}$$

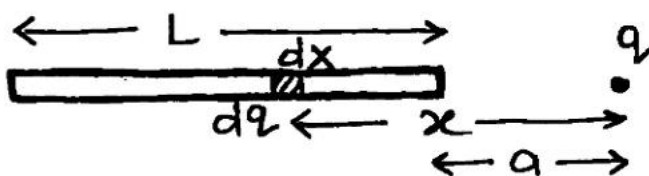


- (b) If a dielectric medium (dielectric Const. K and thickness t) is partially filled between the charges then effective air separation between the charges becomes $(r - t + t\sqrt{K})$

$$F_{\text{medium}} = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{(r - t + t\sqrt{K})^2}$$



Force on Rod due to Point charge



$$dF = \frac{kq \times dq}{x^2}$$

$$\text{or } dF = \frac{kq \lambda dx}{x^2}$$

λ = Linear charge density

$$F = \frac{kq\lambda L}{a(a+L)}$$

Q. The acceleration of an electron due to the mutual attraction between the electron and proton when they are $1.6 \text{ \AA}$ apart is

Sol. Force b/w electron and proton

$$F = \frac{k \cdot e \cdot e}{r^2}$$

$$F = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{(1.6 \times 10^{-10})^2} = 9 \times 10^{-9} \text{ N}$$

then acceleration of electron $a = \frac{F}{m_e}$

$$a = \frac{9 \times 10^{-9}}{9.1 \times 10^{-31}} = 0.98 \times 10^{22} \text{ m/s}^2$$

Q. Two spheres having charges q and $-q$ exert force F on each other. Now 20% charge of one sphere is transferred to another sphere. Then find the new force b/w them.

Sol. Given that $F = \frac{K(q)(-q)}{r^2}$

When 20% charge is transferred from one to another then

$$F' = \frac{K(0.8q)(0.2q - q)}{r^2}$$

$$\text{or } F' = \frac{K(0.8q)(-0.8q)}{r^2} = 0.64 F$$

Q. A particle of mass m , carrying charge q_1 , is revolving around a fixed charge $-q_2$ in a circular path of radius r . Find the time period of revolution and its speed.

Sol. Here force of attraction of $-q_2$ on q_1 , will provide centripetal force to q_1 .

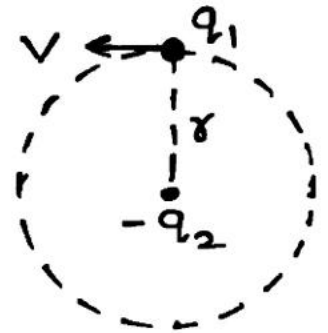
$$\frac{Kq_1q_2}{r^2} = mr\omega^2$$

as $\omega = \frac{2\pi}{T}$ so that $\frac{Kq_1q_2}{r^2} = mr \times \left(\frac{2\pi}{T}\right)^2$

On solving we get $T = 2\pi \sqrt{\frac{mr^3}{Kq_1q_2}}$

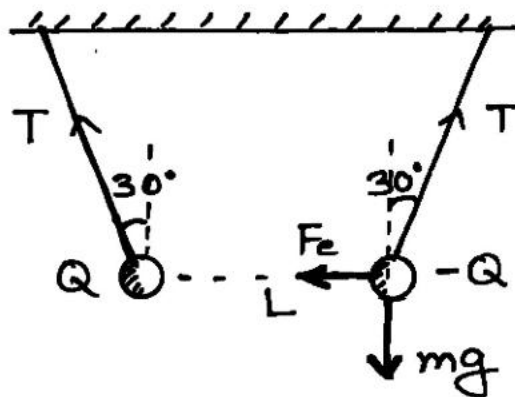
Also we can write $\frac{Kq_1q_2}{r^2} = \frac{mV^2}{r}$

$$V = \sqrt{\frac{Kq_1q_2}{mr}}$$



Q. Find the tension and charge on both bodies having equal and opposite charge.

Sol.



In equilibrium

$$T \cos 30^\circ = mg$$

$$T \times \frac{\sqrt{3}}{2} = mg$$

$$T = \frac{2}{\sqrt{3}} mg$$

Also $T \sin 30^\circ = \frac{KQ \cdot Q}{L^2}$

$$T \times \frac{1}{2} = \frac{KQ^2}{L^2} \Rightarrow Q^2 = \frac{L^2}{2K} \times \frac{2}{\sqrt{3}} mg$$

$$Q = \sqrt{\frac{mgL^2}{\sqrt{3}K}}$$

Q. Two equal positive charges each of $2\mu\text{C}$ interact with a third positive charge of $3\mu\text{C}$ situated as shown in fig. Calculate the magnitude and direction of the force on the $3\mu\text{C}$ charge.

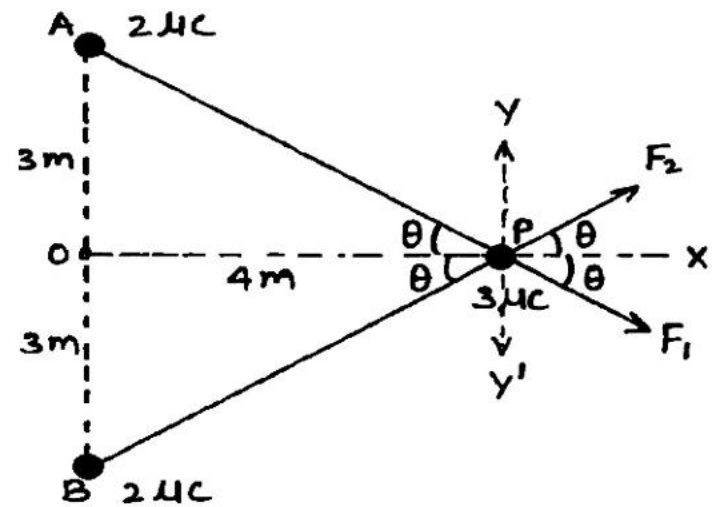
Sol.

$$AP = BP = \sqrt{3^2 + 4^2} = 5\text{ m}$$

$$F_1 = F_2 = \frac{Kq_1q_2}{r^2}$$

$$= \frac{9 \times 10^9 \times 2 \times 10^{-6} \times 3 \times 10^{-6}}{5^2}$$

$$F_1 = F_2 = 2.16 \times 10^{-3} \text{ N}$$



Here the components of F_1 and F_2 along Y and axis are equal in magnitude ($F_1 \sin \theta = F_2 \sin \theta$) but opposite in direction so they cancel each other

$$F = 2 F_1 \cos \theta = 2 \times 2.16 \times 10^{-3} \times \frac{4}{5}$$

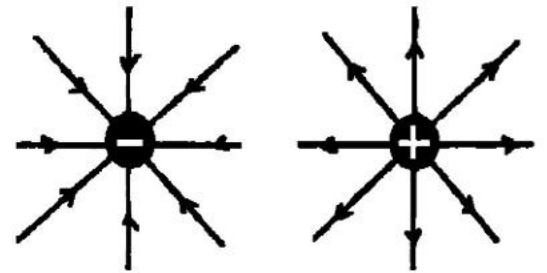
$$\text{So } F = 3.5 \times 10^{-3} \text{ N}$$

Electric Field

An electric field is the region of space surrounding electrically charged particles where a force is exerted on other electrically charged objects.

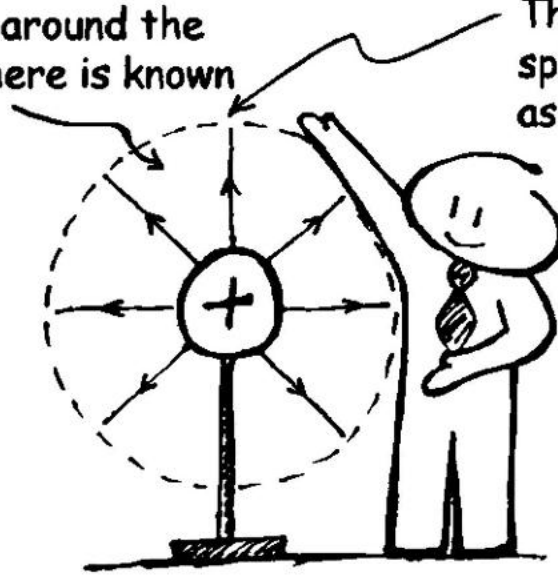
* The direction of the field is taken to be the direction of the force it would exert on a positive test charge.

* The electric field is radially outward from a positive charge and radially inward towards a negative charge.



The space or region around the positive charged sphere is known as electric field.

The lines around the sphere/particle are known as electric field lines.

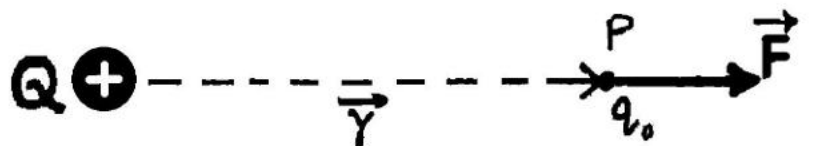


The denser the lines (closer/more lines) the stronger the electric field strength.

Electric Field Intensity ($\vec{E}$)

The electric field intensity at any point is the strength of electric field at that point. It is defined as the force experienced by unit Positive charge placed at that Point.

$$F = \frac{1}{4\pi\epsilon_0} \frac{Q \cdot q_0}{r^2}$$



by definition $\vec{E} = \frac{\vec{F}}{q_0}$ $\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r}$

where $\hat{r}$ is unit vector directed from Q towards q_0 .

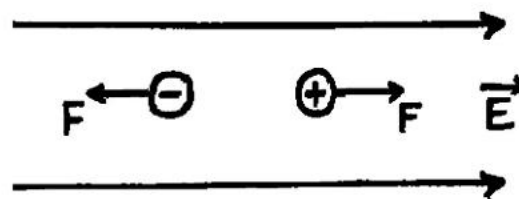
SI Unit - N/C, V/m Dimensions - $[MLT^{-3}A^{-1}]$

Important Points

- (1) The magnitude of electric field $\vec{E}$ is same at equal distances from the charge Q , thus it has a spherical symmetry.
- (2) The test charge q_0 may have its own electric field, it may modify the electric field of the source charge. Therefore to minimize this effect we rewrite electric field intensity at $\vec{r}$ as.

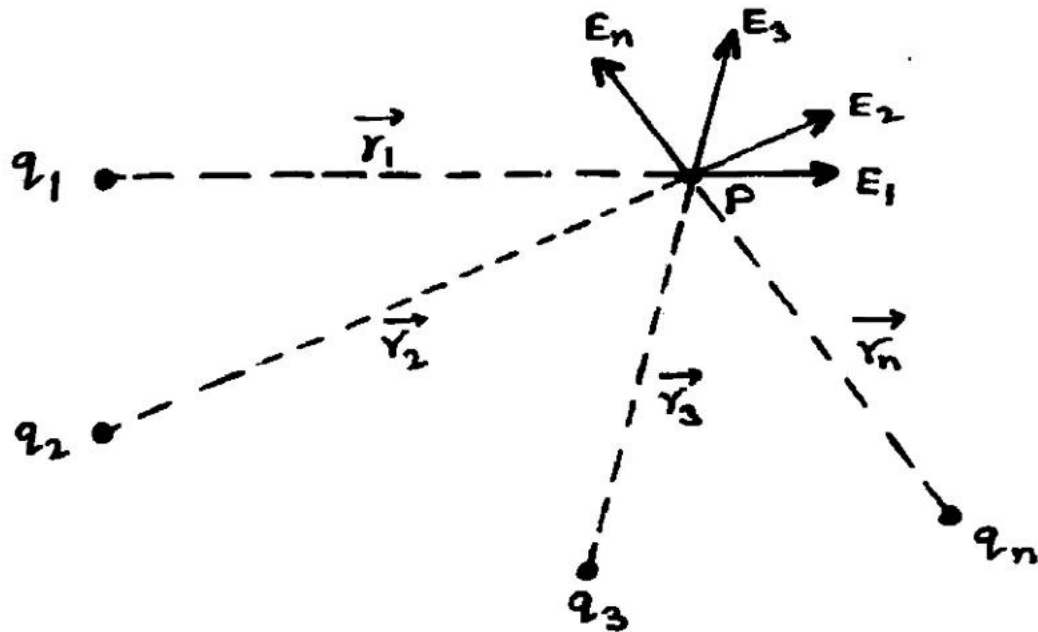
$$\vec{E} = \lim_{q_0 \rightarrow 0} \frac{\vec{F}}{q_0}$$

Note - The direction of force on positive charge is along the direction of electric field and on the negative charge is opposite to the electric field.



Electric Field due to a system of charges

Consider a system of charges $q_1, q_2, \dots, q_n$ with position vectors $\vec{r}_1, \vec{r}_2, \dots, \vec{r}_n$.



The electric field intensity at a point P due to a system of charges is the vector sum of the electric fields at the point due to individual charges.

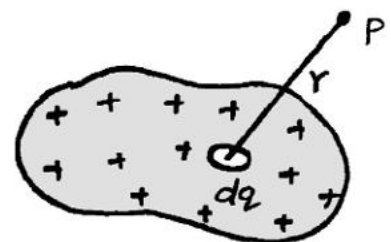
$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \dots + \vec{E}_n$$

$$\vec{E} = \frac{kq_1}{|\vec{r}_1|^2} \hat{r}_1 + \frac{kq_2}{|\vec{r}_2|^2} \hat{r}_2 + \dots + \frac{kq_n}{|\vec{r}_n|^2} \hat{r}_n$$

Note -

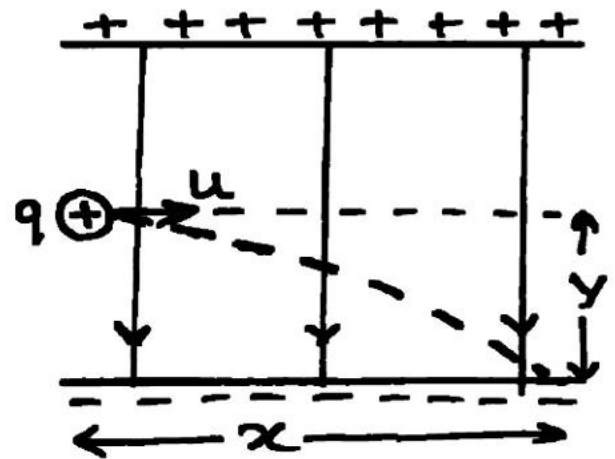
For Continuous charge distribution

$$\vec{E} = K \int \frac{dq}{r^2} \hat{r}$$



Motion of charge in Electric Field

When a charge particle enters perpendicularly in a uniform electric field with velocity u , then it describes a parabolic path.



- Along x-axis there is no acceleration, so that

$$x = ut$$

and $v_x = u$.

- Along y-axis acceleration $a_y = \frac{qE}{m}$

Hence Vertical displacement $y = \frac{1}{2} \left(\frac{qE}{m} \right) t^2$

or $y = \frac{1}{2} \left(\frac{qE}{m} \right) \left(\frac{x}{u} \right)^2$

- Vertical Velocity $v_y = a_y t = \frac{qE}{m} t$

∴ Velocity of particle after time t

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + \left(\frac{qE}{m} t \right)^2}$$

Equilibrium of suspended charge from string

In equilibrium -

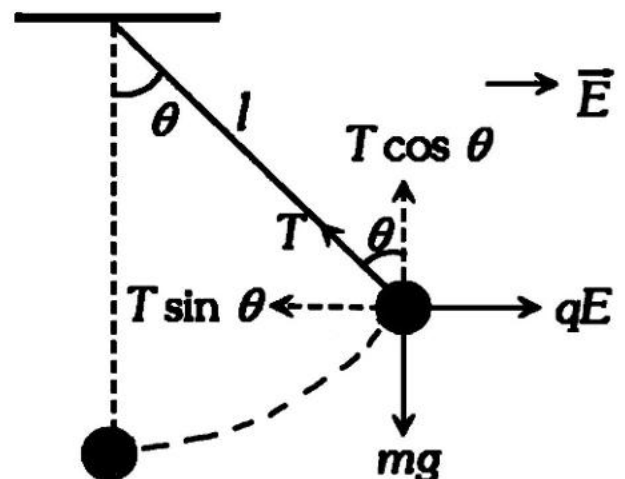
$$T \sin \theta = qE \quad \text{--- (1)}$$

$$T \cos \theta = mg \quad \text{--- (2)}$$

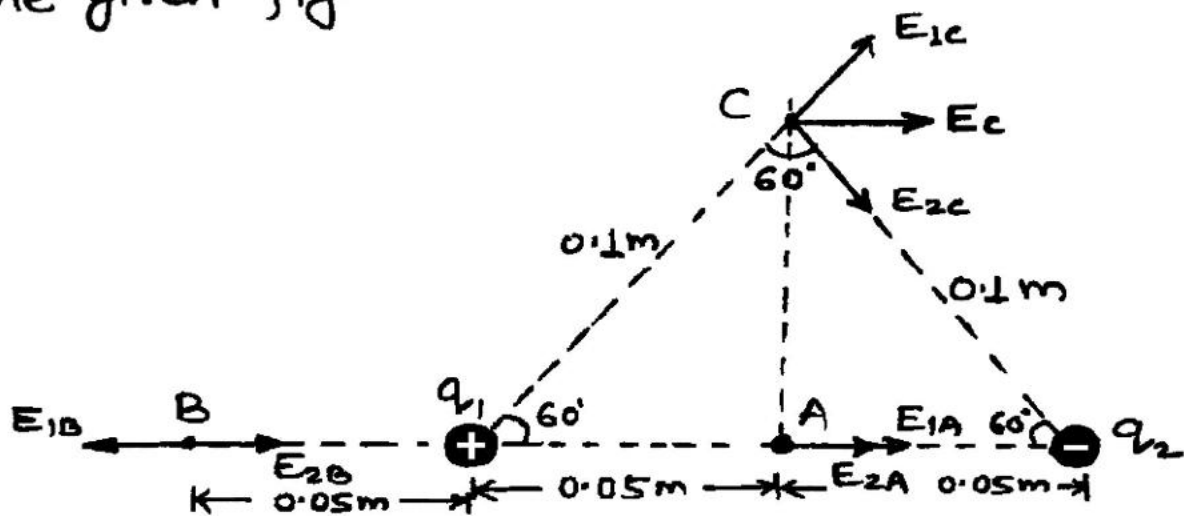
From eq. (1) and (2)

$$T = \sqrt{(qE)^2 + (mg)^2}$$

and $\tan \theta = \frac{qE}{mg}$



Q. Two point charges q_1 and q_2 of magnitude 10^{-8}C and -10^{-8}C respectively are placed 0.1m apart. Calculate the electric field at points A, B and C in the given fig.



Electric field at point A due to charge q_1

$$E_{1A} = \frac{9 \times 10^9 \times 10^{-8}}{(0.05)^2} = 3.6 \times 10^4 \text{ N/C (Towards right)}$$

Electric field at point A due to charge q_2

$$E_{2A} = \frac{9 \times 10^9 \times 10^{-8}}{(0.05)^2} = 3.6 \times 10^4 \text{ N/C (Towards right)}$$

Total Electric field at point A = $E_A = E_{1A} + E_{2A}$

$$E_A = 7.2 \times 10^4 \text{ N/C (Towards the right)}$$

Electric field at point B due to charge q_1

$$E_{1B} = \frac{9 \times 10^9 \times 10^{-8}}{(0.05)^2} = 3.6 \times 10^4 \text{ N/C (Towards left)}$$

Electric field at point B due to charge q_2

$$E_{2B} = \frac{9 \times 10^9 \times 10^{-8}}{(0.15)^2} = 4 \times 10^3 \text{ N/C (Towards right)}$$

Total Electric field at point B = $E_B = E_{1B} - E_{2B}$

$$E_B = 3.6 \times 10^4 - 4 \times 10^3 = 3.2 \times 10^4 \text{ N/C (Towards Left)}$$

The electric field at point C due to charge q_1 and q_2 are equal in magnitude

$$E_{1c} = E_{2c} = \frac{9 \times 10^9 \times 10^{-8}}{(0.1)^2} = 9 \times 10^3 \text{ N/C}$$

The resultant electric field at point C is

$$E_c = \sqrt{E_{1c}^2 + E_{2c}^2 + 2 E_{1c} E_{2c} \cos 120^\circ}$$

$$E_c = \sqrt{E_{1c}^2 + E_{1c}^2 + 2 E_{1c} E_{1c} \times \left(-\frac{1}{2}\right)} \quad [\because E_{1c} = E_{2c}]$$

$$E_c = E_{1c} = 9 \times 10^3 \text{ N/C} \quad [\text{Towards right}]$$

Q. What are the magnitude and direction of the electric field at centre of the square in figure. If $q = 10^{-8} \text{ C}$ and $a = 5 \text{ cm}$.

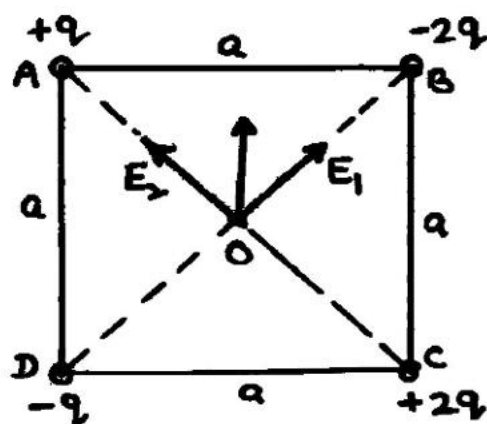
Sol. Here

$$OA = OB = OC = OD = r$$

$$r = \frac{5 \times 10^{-2}}{\sqrt{2}} \text{ m.}$$

also here $E_1 = E_2 = \frac{q}{4\pi\epsilon_0 r^2}$

$$\text{or } E_1 = E_2 = \frac{10^{-8} \times 9 \times 10^9}{\left(\frac{5 \times 10^{-2}}{\sqrt{2}}\right)^2} = 7.2 \times 10^4 \text{ N/C}$$



In square ABCD the $\angle AOB = 90^\circ$

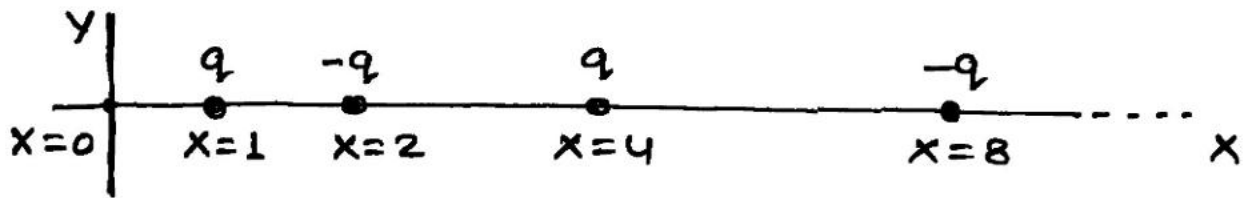
$$\text{Net Electric field at O} = \sqrt{E_1^2 + E_2^2 + 2 E_1 E_2 \cos 90^\circ}$$

$$E = \sqrt{E_1^2 + E_2^2} = E_1 \sqrt{2}$$

$$E = 7.2 \sqrt{2} \times 10^4 \text{ N/C at } 45^\circ \text{ to OA.}$$

Q. An infinite number of charges each q are placed on the x -axis at distance of $1, 2, 4, 8 \dots$ meter from the origin. If the charges are alternately positive and negative then find the intensity of electric field at origin.

Sol.



$$E_{\text{net}} = E_1 - E_2 + E_3 - E_4 + \dots$$

$$E_{\text{net}} = \frac{q}{4\pi\epsilon_0} \left[1 - \frac{1}{2^2} + \frac{1}{4^2} - \frac{1}{8^2} + \dots \right]$$

$$E_{\text{net}} = \frac{q}{4\pi\epsilon_0} \times \left[1 - \frac{1}{4} + \frac{1}{16} - \frac{1}{64} + \dots \right]$$

Since the expression in the bracket is infinite GP. with common ratio $-\frac{1}{4}$ So that

$$E_{\text{net}} = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{1 - (-\frac{1}{4})} \right]$$

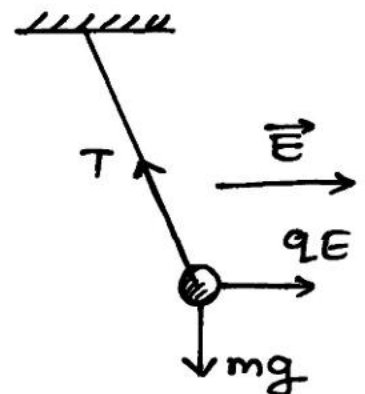
$$E_{\text{net}} = \frac{4}{5} \frac{q}{4\pi\epsilon_0} \Rightarrow E_{\text{net}} = \frac{q}{5\pi\epsilon_0}$$

Q. A pendulum bob of mass $30.7 \times 10^{-6} \text{ kg}$ and carrying a charge $2 \times 10^{-8} \text{ C}$ is at rest in a horizontal uniform electric field of 20000 V/m . Find the tension in the string.

Sol. Tension $T = \sqrt{(mg)^2 + (qE)^2}$

$$T = \sqrt{(30.7 \times 10^{-6} \times 10)^2 + (2 \times 10^{-8} \times 20000)^2}$$

$$T = 5 \times 10^{-4} \text{ N}$$



Q. A charge particle of $1 \mu\text{C}$ is placed at origin, find intensity of electric field due to the charge at position $(3, 4) \text{ m}$.

Sol. Position vector of particle $\vec{r} = 3\hat{i} + 4\hat{j}$

Magnitude of $\vec{r} = \sqrt{(3)^2 + (4)^2} = 5 \text{ m}$

Electric field $\vec{E} = \frac{Kq}{r^2} \hat{r}$

$$\vec{E} = \frac{9 \times 10^9 \times 10^{-6}}{(5)^2} \times \frac{3\hat{i} + 4\hat{j}}{5}$$

$$\vec{E} = 72(3\hat{i} + 4\hat{j}) \text{ N/C}$$

Q. Find intensity of electric field at point P.

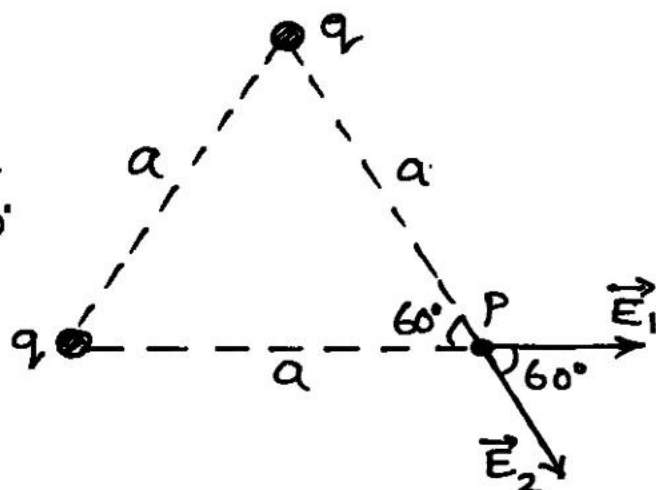
Sol. $E_1 = E_2 = \frac{Kq}{a^2}$

Resultant electric Field

$$E_{\text{net}} = \sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos 60^\circ}$$

$$E_{\text{net}} = \sqrt{E_1^2 + E_1^2 + 2E_1^2 \times \frac{1}{2}}$$

$$E_{\text{net}} = \sqrt{3} E_1 = \sqrt{3} \frac{Kq}{a^2}$$



Q. Calculate the magnitude of an electric field which can just suspend a deuteron of mass $3.2 \times 10^{-27} \text{ kg}$ freely in air.

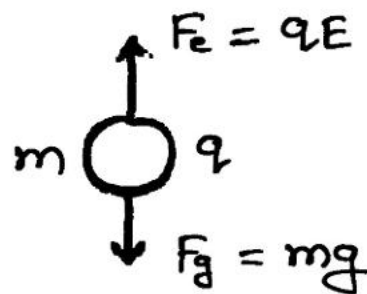
Sol. In equilibrium

$$qE = mg$$

$$E = \frac{mg}{q}$$

$$E = \frac{3.2 \times 10^{-27} \times 10}{1.6 \times 10^{-19}}$$

$$E = 19.6 \times 10^{-8} \text{ N/C}$$



Q. A particle of mass m and charge q is placed at rest in a uniform electric field E and then released. The K.E. attained by the particle after moving a distance y is.

Sol. Acceleration of the particle $a = \frac{F}{m} = \frac{qE}{m}$

$$\text{As } v^2 = u^2 + 2ay$$

$$v^2 = 0 + 2 \times \left(\frac{qE}{m} \right) y \Rightarrow v^2 = \frac{2qEy}{m}$$

$$\text{K.E.} = \frac{1}{2}mv^2$$

$$= \frac{1}{2}m \times \frac{2qEy}{m} = qEy$$

Q. A pendulum has mass 4 mg and carries a charge $2 \times 10^{-9} \text{ C}$. It hangs in equilibrium from a massless thread of length 50 cm . A horizontal electric field of 200 V/m exists in space. Find

- (a) Angle made by thread with the vertical
(b) Tension in the thread

Sol. (a) At equilibrium

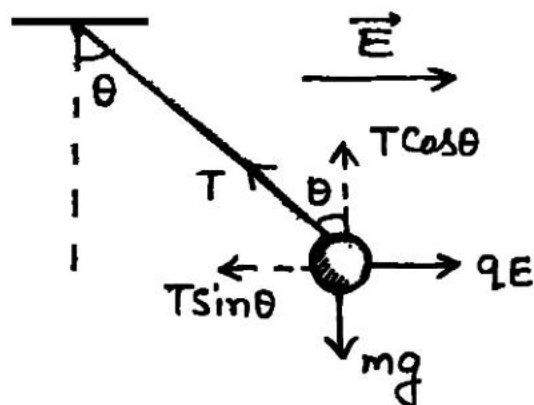
$$T \sin \theta = qE$$

$$T \cos \theta = mg$$

$$\text{Hence } \tan \theta = \frac{qE}{mg}$$

$$\tan \theta = \frac{2 \times 10^{-9} \times 2 \times 10^4}{4 \times 10^{-6} \times 10}$$

$$\tan \theta = 1 \Rightarrow \theta = 45^\circ$$



$$(b) \quad T = \sqrt{(qE)^2 + (mg)^2}$$

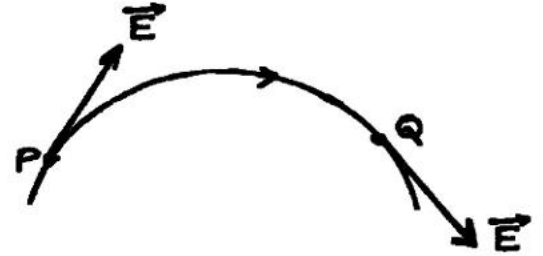
$$T = \sqrt{2(qE)^2} \quad \text{as } qE = mg$$

$$T = \sqrt{2} \times 2 \times 10^{-9} \times 2 \times 10^4 = 5.64 \times 10^{-5} \text{ N}$$

Electric Field Lines

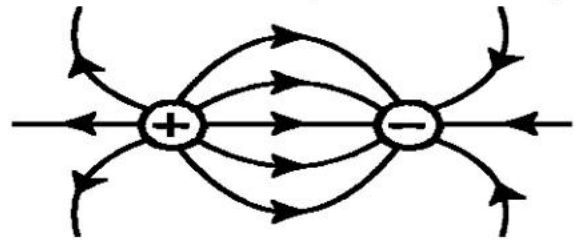
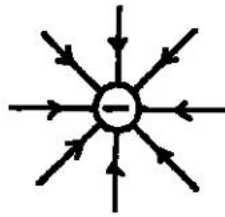
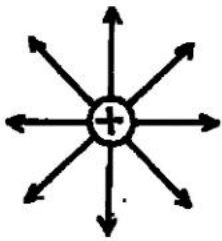
An electric field line is a curve drawn in such a way that the tangent to it at each point is in the direction of the net electric field at that point.

The magnitude of electric field is represented by the density of field lines.

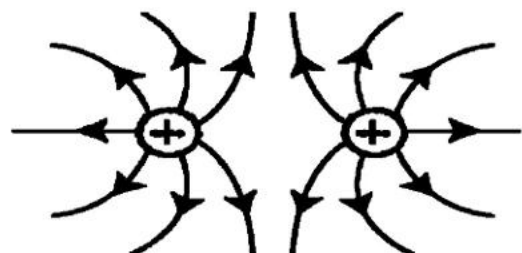
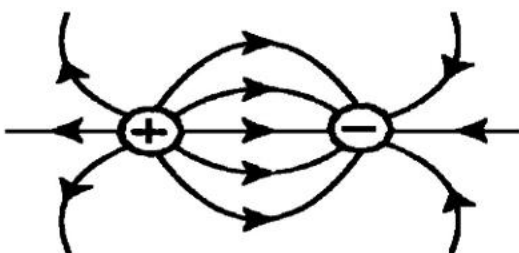


Properties

- 1.] Imaginary & Continuous curves.
- 2.] Starts from positive charge and end at negative charge.



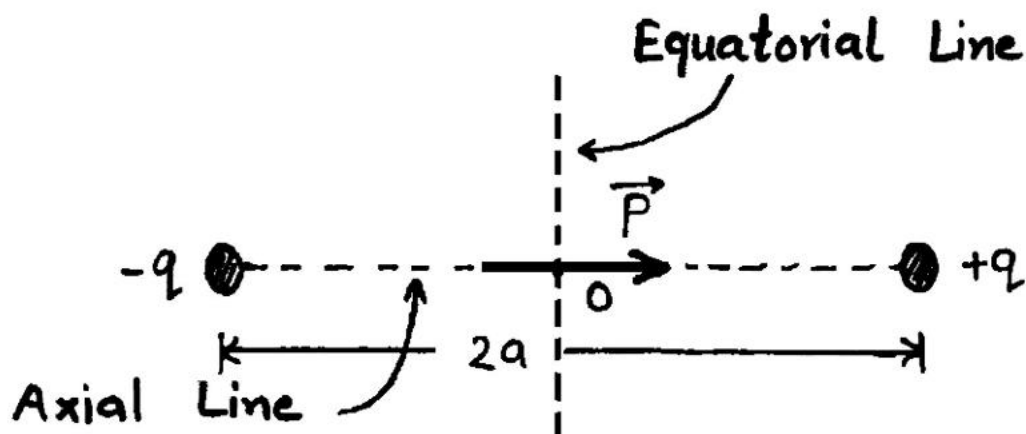
- 3.] Do not form continuous closed loops.
- 4.] Always normal to the surface of the conductor.
- 5.] Can never cross each other.
- 6.] Lines of force per unit area passing normally at a point represents magnitude of electric field.
- 7.] Around a system of two positive charges field lines gives pictorial representation of their mutual repulsion.
- 8.] Around a system of two negative charges field lines shows the mutual attraction.



Electric Dipole

An electric dipole is a pair of equal and opposite point charges separated by a short distance.

Let q and $-q$ are two equal and opposite point charges separated by a small distance ' $2a$ '.



* The total charge of the electric dipole is zero, but this does not mean that the field of the electric dipole is zero.

Dipole Moment ($\vec{P}$)

Dipole moment is a measure of the strength of electric dipole. It is a vector quantity whose direction is from negative charge to positive charge.

Dipole moment is equal to the product of the magnitude of either charge and the distance between them.

$$\vec{P} = q \times 2a \hat{p}$$

SI Unit of Electric Dipole Moment- Coulomb.meter

Dipole's Electric field

(A) On axial line

Electric field at P due to positive charge $E_{+q} = \frac{Kq}{(r-a)^2}$

Electric field at P due to negative charge $E_{-q} = \frac{Kq}{(r+a)^2}$

Net Electric field E at P

$$E = E_{+q} - E_{-q}$$

$$E = \frac{Kq}{(r-a)^2} - \frac{Kq}{(r+a)^2}$$

$$E = \frac{Kq \times 4ra}{(r^2 - a^2)^2}$$

$$E = \frac{2KPr}{(r^2 - a^2)^2}$$



$$[\because P = q \times 2a]$$

If $r \gg a$ then $E = \frac{2KP}{r^3}$

 $\Rightarrow$

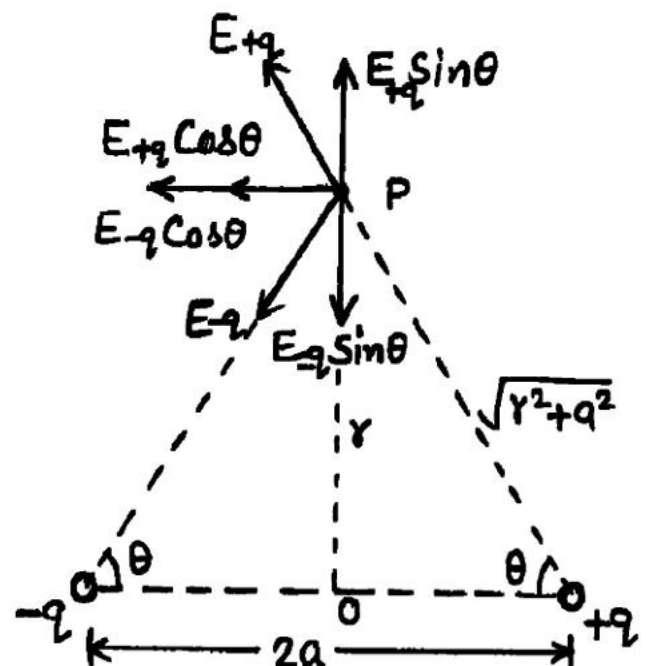
$$\vec{E} = \frac{2K\vec{P}}{r^3}$$

(B) On Equatorial Line

Electric Field due to positive charge $E_{+q} = \frac{Kq}{(r^2 + a^2)}$

Electric Field due to negative charge $E_{-q} = \frac{Kq}{(r^2 + a^2)}$

Vertical Component of E_{+q} and E_{-q} will cancel each other and horizontal components will be added.



So net electric field at P is $E = E_{+q} \cos \theta + E_{-q} \cos \theta$

$$E = 2 E_{+q} \cos \theta \quad [\because E_{+q} = E_{-q}]$$

$$E = \frac{2 K q \cos \theta}{(r^2 + a^2)} = \frac{2 K q a}{(r^2 + a^2)^{3/2}} \quad \left[\because \cos \theta = \frac{a}{\sqrt{r^2 + a^2}} \right]$$

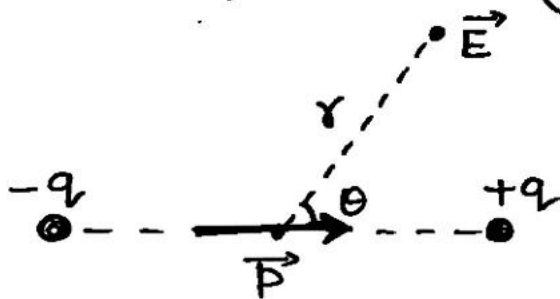
If $r \gg a$ then $E = \frac{K p}{r^3}$

or $\boxed{\vec{E} = -\frac{K \vec{P}}{r^3}}$

Note- * Hence the dipole field at large distances falls off not as $1/r^2$ but as $1/r^3$.

* Due to an electric dipole $E_{\text{axial}} = 2 E_{\text{equatorial}}$

* Electric field at angle θ from dipole moment



$$\vec{E} = \frac{K \vec{P}}{r^3} \sqrt{3 \cos^2 \theta + 1}$$

Angle between direction of $\vec{E}$ and $\vec{P} = \theta + \alpha$

where $\tan \alpha = \frac{\tan \theta}{2}$



Dipole in a uniform Electric Field

Net Force $F_{\text{net}} = 0$

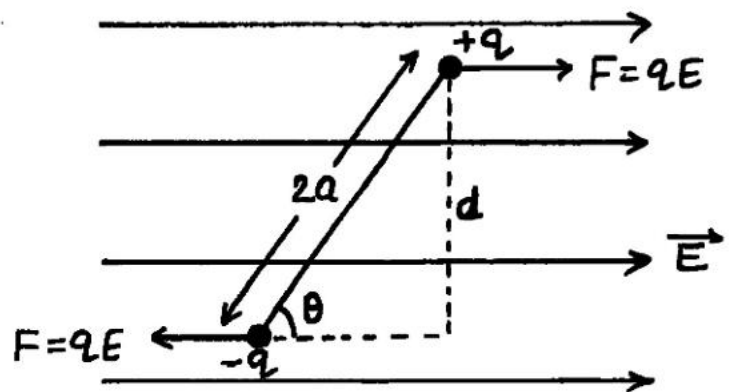
Torque acting on dipole

$$\tau = F \times d$$

$$\tau = qE \times 2a \sin \theta$$

$$\tau = pE \sin \theta$$

$$\vec{\tau} = \vec{p} \times \vec{E}$$



Case 1. When $\vec{p}$ is along $\vec{E}$, $\theta = 0$ then $\tau = 0$
Dipole is in *stable Equilibrium*.

Case 2. When $\vec{p}$ is opposite $\vec{E}$, $\theta = 180^\circ$ then $\tau = 0$
Dipole is in *unstable Equilibrium*.

Note- If $\vec{E}$ is nonuniform then the dipole will experience both a resultant force and a torque, so its motion will be combined translatory and rotatory.

Oscillation of a Dipole in a Uniform $\vec{E}$

When a dipole is suspended in a uniform $\vec{E}$, it will align itself parallel to the field. Now if it is given a small angular displacement θ about its equilibrium position then the restoring torque will be $\tau = pE \sin \theta$

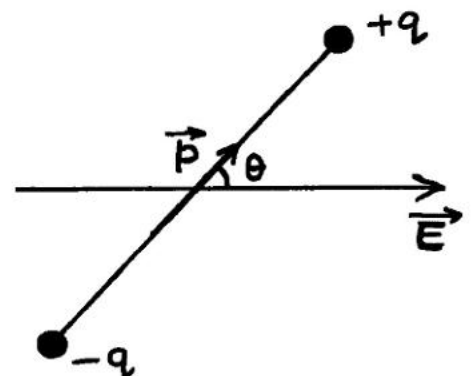
If θ is small then $\tau = pE \theta$

But $\tau = I\alpha$ so that

$$I\alpha = pE \theta \quad \Rightarrow \quad \alpha = \frac{pE}{I} \theta$$

$$\text{or } \frac{pE}{I} \theta = \omega^2 \theta \quad [\text{Angular S.H.M.}]$$

$$\text{then } \omega = \sqrt{\frac{pE}{I}} \quad \Rightarrow \quad T = 2\pi \sqrt{\frac{I}{pE}}$$



Q. In a certain region of space, electric field is along the z-direction. The magnitude of electric field is not constant and increases uniformly along positive z-axis, at the rate of 10^5 N/C per metre. Calculate the Force and torque experienced by a system having a total dipole moment 10^{-7} C.m. in the negative z direction?

Sol. Dipole moment $|\vec{P}| = q \times dL = -10^{-7} \text{ C.m}$

Rate of increase of electric Field per unit length,

$$\frac{dE}{dL} = 10^5 \text{ N/C per meter}$$

Force experienced by the system is given by

$$F = qE$$

$$\text{or } F = q \times dL \times \frac{dE}{dL}$$

$$F = -10^{-7} \times 10^5 = -10^{-2} \text{ N (along -z axis)}$$

Here the angle between electric field and dipole moment is 180° .

$$\text{So Torque } \tau = PE \sin \theta$$

$$\tau = PE \sin 180^\circ = 0$$

Q. Find the torque on a dipole of dipole moment $10^{-7} (5\hat{i} + \hat{j} - 2\hat{k}) \text{ C.m}$ in an electric field $10^7 (\hat{i} + \hat{j} + \hat{k}) \frac{\text{N}}{\text{C}}$

Sol.

$$\vec{\tau} = \vec{p} \times \vec{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 1 & -2 \\ 1 & 1 & 1 \end{vmatrix}$$

$$\vec{\tau} = \hat{i} [1 - (-2)] - \hat{j} [5 - (-2)] + \hat{k} [5 - 1]$$

$$\vec{\tau} = 3\hat{i} - 7\hat{j} + 4\hat{k}$$

$$|\vec{\tau}| = \sqrt{(3)^2 + (-7)^2 + (4)^2} = 8.6 \text{ N.m}$$

Q. Find the electric field due to a dipole moment $2 \times 10^{-8} \text{ C.m}$ at a point distance 1 m from the centre of dipole, when line joining the point to the centre of dipole makes an angle of 60° with the dipole axis.

Sol. Given $p = 2 \times 10^{-8} \text{ C.m}$, $r = 1 \text{ m}$, $\theta = 60^\circ$

$$E = \frac{kp}{r^3} \sqrt{3\cos^2\theta + 1}$$

$$E = \frac{9 \times 10^9 \times 2 \times 10^{-8}}{(1)^3} \sqrt{3 \times \cos^2 60^\circ + 1}$$

$$E = 238 \text{ N/C}$$

Q. An electric dipole is placed at an angle of 30° with an electric field $2 \times 10^5 \text{ N/C}$. It experiences a torque equal to 4 N.m . Find the charge on the dipole if dipole length is 2 cm .

Sol. Given - $\tau = 4 \text{ N.m}$, $E = 2 \times 10^5 \text{ N/C}$, $2a = 2 \text{ cm}$.

$$\tau = p E \sin\theta$$

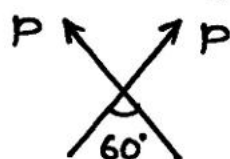
$$\text{or } \tau = (q \times 2a) E \sin\theta$$

$$q = \frac{\tau}{2a E \sin\theta}$$

$$q = \frac{4}{2 \times 10^{-2} \times 2 \times 10^5 \sin 30^\circ} = 2 \times 10^{-3} \text{ C}$$

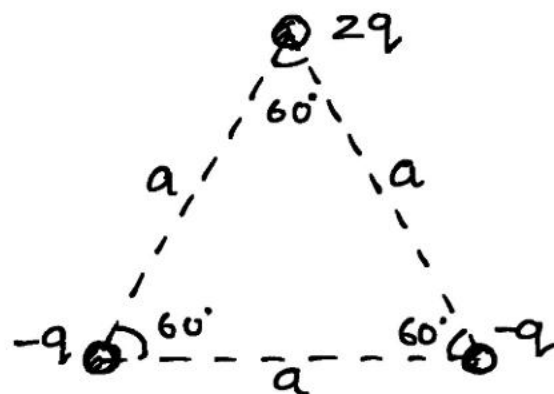
Q. Find the electric dipole moment of given system.

Sol.



$$P_{\text{net}} = \sqrt{P^2 + P^2 + 2P \cdot P \cos 60^\circ}$$

$$= \sqrt{3} P = \sqrt{3} q \times a$$

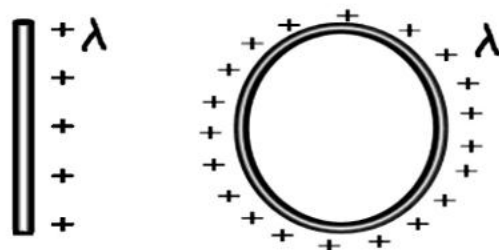


Continuous distribution of charge

1.] Linear Charge distribution

Linear charge density $\lambda = \frac{Q}{l}$

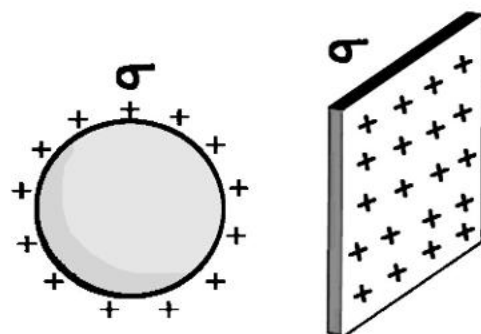
Example - Charged wire, ring etc.



2.] Surface Charge distribution

Surface Charge density $\sigma = \frac{Q}{A}$

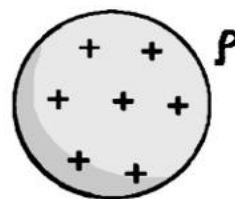
Example - Charged plane sheet, sphere etc.



3.] Volume Charge distribution

Volume Charge density $\rho = \frac{Q}{V}$

Example - charged insulating sphere etc.

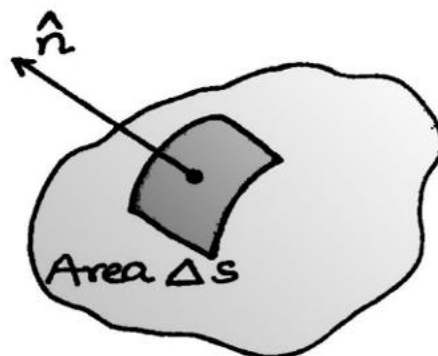


Area Vector

By convention, the vector associated with every area element of a closed surface is taken to be in the direction of the outward normal.

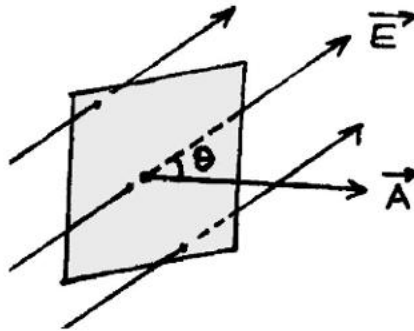
$$\Delta \vec{S} = (\Delta s) \hat{n}$$

Here Δs is magnitude of the area element and $\hat{n}$ is a unit vector in the direction of outward normal at that point.



Electric flux (Φ)

It is a measure of flow of electric field through a surface. It is equal to the product of an area element and the perpendicular component of electric field ($\vec{E}$) integrated over a surface.



For open surface $\left\{ \begin{array}{l} \rightarrow \Phi = \vec{E} \cdot \vec{A} \text{ (for plane surface)} \\ \rightarrow \Phi = \int \vec{E} \cdot d\vec{S} \text{ (for nonuniform surface)} \end{array} \right.$

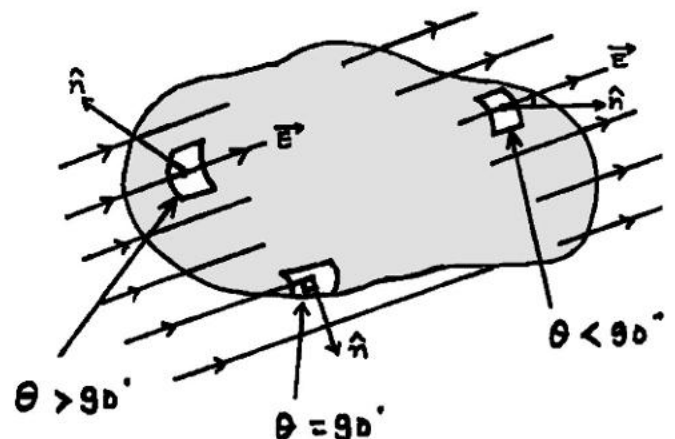
For closed surface $\left\{ \begin{array}{l} \rightarrow \Phi = \oint \vec{E} \cdot d\vec{S} \text{ (by definition)} \\ \rightarrow \Phi = \frac{q}{\epsilon_0} \text{ (by Gauss's Law)} \end{array} \right.$

SI Unit - V.m. or Nm^2/C

Dimensions - $[M^1 L^3 T^{-3} A^{-1}]$

Note- The value of Φ does not depend upon the distribution of charges and the distance between them inside the closed surface.

For a closed surface outward flux is +ve and inward flux is -ve.



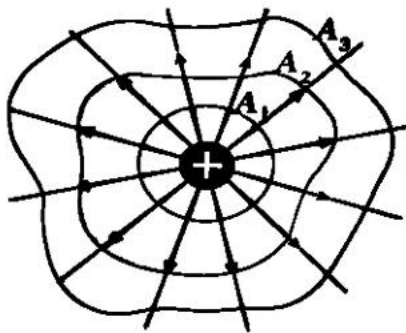
Gauss's Law

The total flux linked with a closed surface is $\frac{1}{\epsilon_0}$ times the charge enclosed by the closed surface.

$$\Phi = \oint \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0}$$

Important facts about Gauss's Law

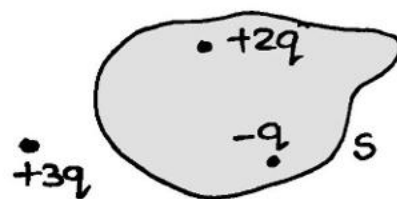
- (i) Gauss's law is valid only for the fields, which follow coulomb's 'inverse square law'.
- (ii) According to Gauss's Law, electric flux passing through a closed surface doesn't depend on shape of surface.



Net Electric Flux is the same through all surfaces.

- (iii) Electric flux passing through a closed surface doesn't depend on charge distribution or location of charge inside the surface.
- (iv) Net electric flux depends on value and nature of the charge and medium.
- (v) If a symmetric body has n identical faces and charge q is located at its centre then electric flux associated with each of its face is $\Phi/n = q/n\epsilon_0$.
- (vi) Net electric flux associated with a closed surface placed in an electric field (uniform or nonuniform) is zero, if there is no charge inside the surface.

Q. In the given fig. find the electric flux due to the charges $+2q$, $-q$ and $+3q$, through the surface 's'.



Sol. Electric Flux = $\frac{1}{\epsilon_0} \times (\text{Total charge within the surface})$

$$\phi = \frac{1}{\epsilon_0} \times (2q - q) = \frac{q}{\epsilon_0}$$

Q. The inward and outward electric flux for a closed surface is $8 \times 10^3 \text{ Nm}^2/\text{C}$ and $4 \times 10^3 \text{ Nm}^2/\text{C}$. Then the total charge inside the surface is

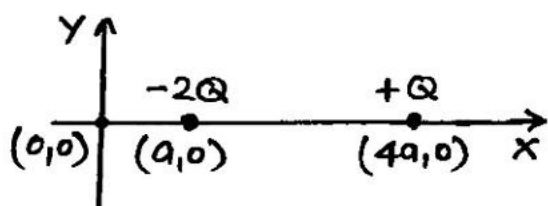
Sol. By Gauss's Law $\phi = \frac{q_{\text{in}}}{\epsilon_0}$

$$q_{\text{in}} = \phi \times \epsilon_0$$

$$q_{\text{in}} = (-8 \times 10^3 + 4 \times 10^3) \times 8.85 \times 10^{-12} \\ = -3.54 \times 10^{-8} \text{ C}$$

Q. Two charges of magnitudes $-2Q$ and $+Q$ are located at points $(a, 0)$ and $(4a, 0)$ respectively. What is the electric flux due to these charges through a sphere of radius ' $3a$ ' with its centre at the origin?

Sol.

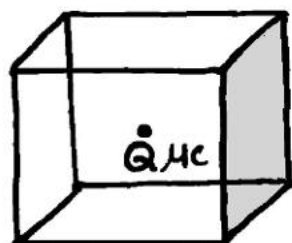


$$\text{Total Electric flux } \phi = \frac{q_{\text{in}}}{\epsilon_0}$$

$$\phi = \frac{-2Q}{\epsilon_0}$$

Q. A charge $Q \mu\text{C}$ is placed at the centre of a cube. What would be the flux through one face?

Sol.



Electric flux through the whole cube

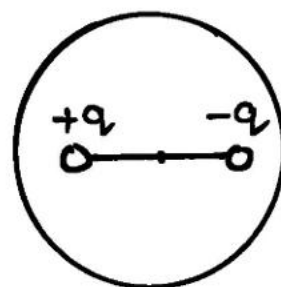
$$\phi = \frac{Q}{\epsilon_0}$$

Electric flux through one face

$$\phi' = \frac{1}{6} \frac{Q}{\epsilon_0}$$

Q. A sphere encloses an electric dipole with charge $\pm 3 \times 10^{-6} \text{ C}$. What is the total electric flux across the sphere?

Sol. by Gauss's law $\phi = \frac{q_{\text{in}}}{\epsilon_0}$
 Net charge of a dipole = 0
 So that $\phi = \frac{0}{\epsilon_0} = 0$



Q. A hollow cylinder has a charge q within it. If ϕ is the electric flux associated with the curved surface B then the flux linked with the plane surface A will be

Sol. by Gauss's Law

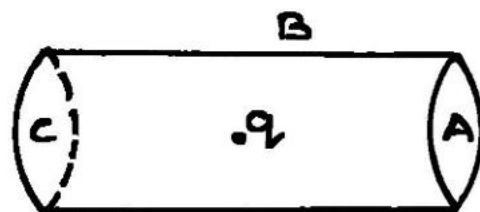
$$\phi_{\text{net}} = \frac{q_{\text{in}}}{\epsilon_0}$$

$$\phi_A + \phi_B + \phi_C = \frac{q}{\epsilon_0}$$

Since $\phi_A = \phi_C$ so that

$$2\phi_A + \phi_B = \frac{q}{\epsilon_0} \Rightarrow 2\phi_A = \frac{q}{\epsilon_0} - \phi$$

$$\phi_A = \frac{1}{2} \left(\frac{q}{\epsilon_0} - \phi \right)$$



Electric Field due to special Charge distribution

1.] An infinite Line distribution of charge

From Gauss's theorem

$$\oint \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0}$$

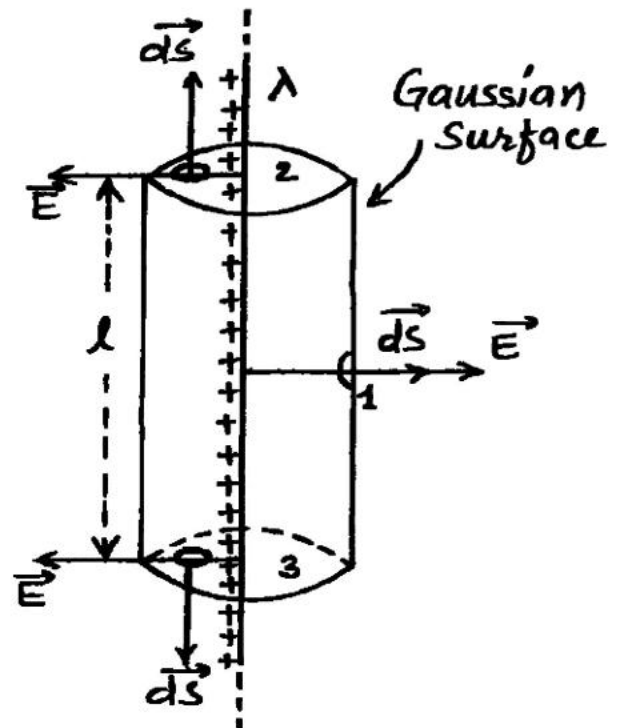
$$\int_1 \vec{E} \cdot d\vec{s} + \int_2 \vec{E} \cdot d\vec{s} + \int_3 \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0}$$

$$\therefore \int_1 E ds + 0 + 0 = \frac{q}{\epsilon_0}$$

$$E \times 2\pi r l = \frac{q}{\epsilon_0}$$

$$E \times 2\pi r l = \frac{\lambda l}{\epsilon_0}$$

$$\text{or } E = \frac{\lambda}{2\pi\epsilon_0 r} \Rightarrow \boxed{E = \frac{2K\lambda}{r}}$$



2.] An Conducting or Hollow sphere

(i) For Outside point -

Using Gauss's theorem

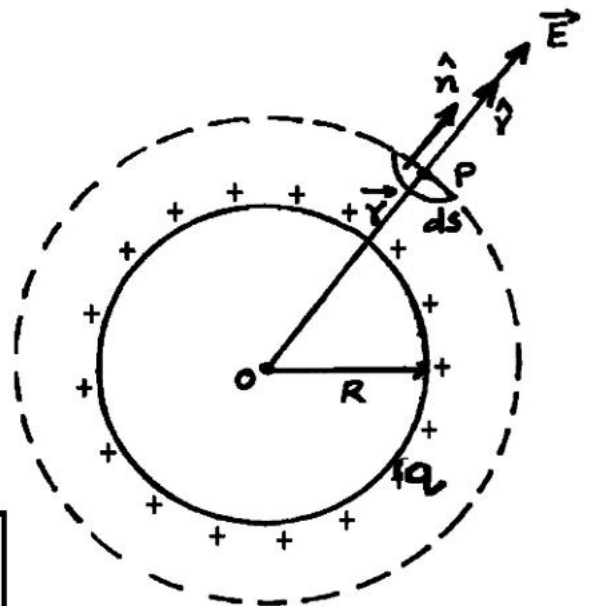
$$\oint \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0}$$

$$\oint E ds \cos 0^\circ = \frac{q}{\epsilon_0}$$

$$E \oint ds = \frac{q}{\epsilon_0}$$

$$E \times 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \Rightarrow \boxed{E = \frac{Kq}{r^2}}$$



(ii) For surface point $r = R$

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{R^2} \Rightarrow \boxed{E = \frac{Kq}{R^2}}$$

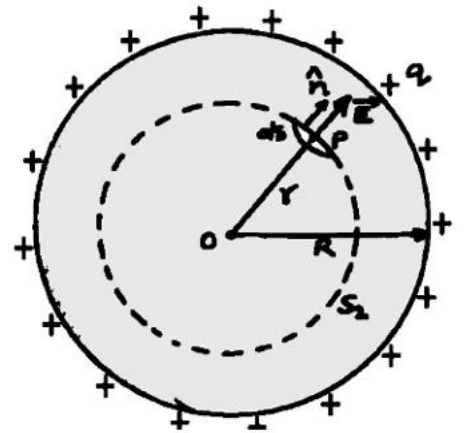
(iii) For Inside point ($r < R$)

Because charge inside the conducting or hollow sphere is zero.

$$\oint \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0} = 0$$

Hence

$$\boxed{E = 0}$$

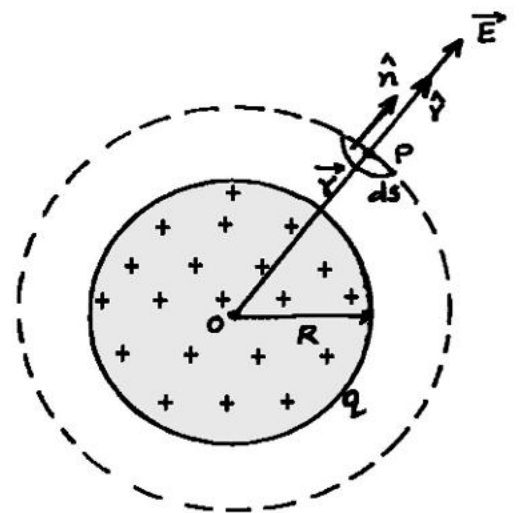
3.] An solid nonconducting sphere(i) Outside Point ($r > R$)

From Gauss's theorem

$$\oint \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0}$$

$$E \times 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$\text{or } E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \Rightarrow \boxed{E = \frac{kq}{r^2}}$$

(ii) At surface ($r = R$)

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{R^2} \Rightarrow \boxed{E = \frac{kq}{R^2}}$$

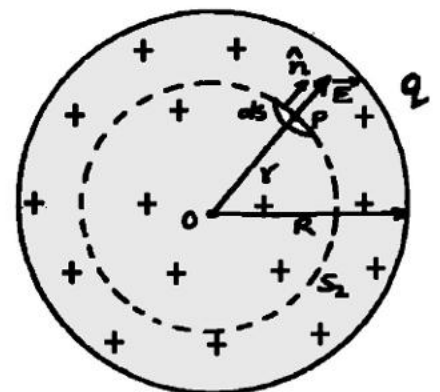
(iii) Inside Point ($r < R$)

From Gauss's theorem

$$\oint \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0}$$

$$E \times 4\pi r^2 = \frac{1}{\epsilon_0} \times \left(\frac{q r^3}{R^3} \right)$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{q r}{R^3} \Rightarrow \boxed{E = \frac{kq r}{R^3}}$$



4.] At the axis of a charged Ring

Let a small element dl have charge $dq = \lambda \times dl = \frac{q}{2\pi a} \times dl$

Electric field due to this charge at P,
 $dE = \frac{dq}{4\pi\epsilon_0 r^2}$

Another diametrically opposite element of length dl also produces electric field dE .

The vertical components will cancel each other and horizontal components will be added.

So net electric field $E = \sum dE \cos \theta = \int dE \cos \theta$

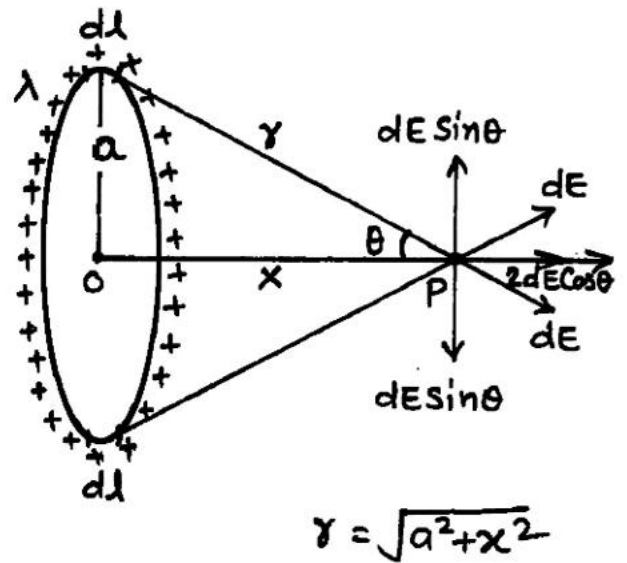
$$E = \int \frac{dq}{4\pi\epsilon_0 r^2} \times \frac{x}{r} \quad \Rightarrow \quad E = \int \frac{q dl}{2\pi a} \times \frac{1}{4\pi\epsilon_0 r^3} \times x$$

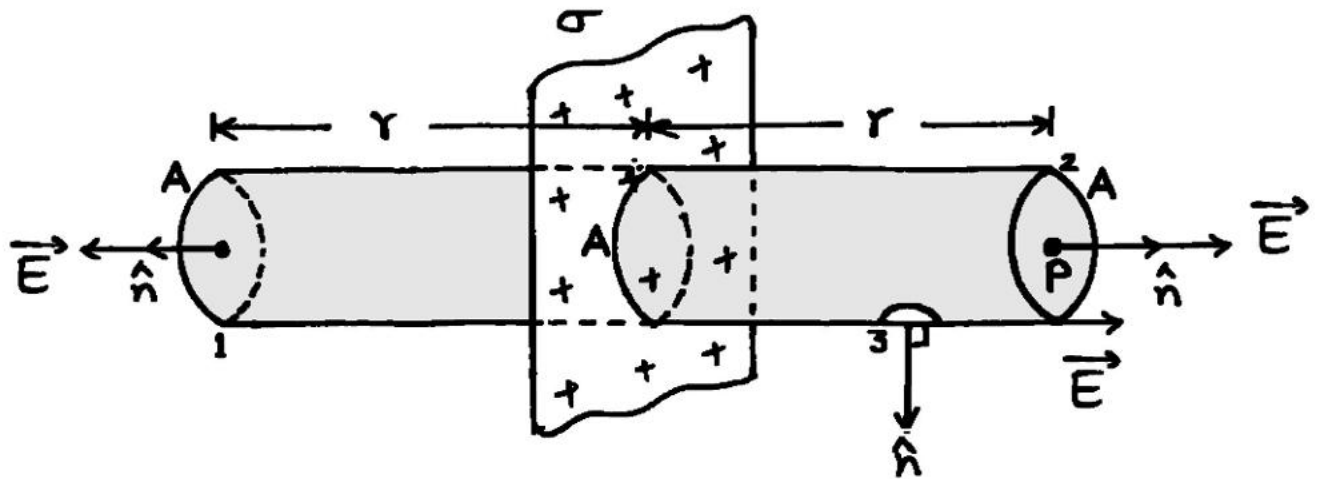
$$E = \frac{q}{2\pi a} \times \frac{x}{4\pi\epsilon_0 r^3} \int dl \quad \Rightarrow \quad E = \frac{q}{2\pi a} \times \frac{x}{4\pi\epsilon_0 r^3} \times 2\pi a$$

$$E = \frac{qx}{4\pi\epsilon_0 r^3} \Rightarrow \boxed{E = \frac{qx}{4\pi\epsilon_0 (a^2 + x^2)^{3/2}} = \frac{kqx}{(a^2 + x^2)^{3/2}}}$$

* At centre ($x=0$) $E=0$

* If x increases Electric field E due to ring first $\uparrow$ then $\downarrow$ and at $x = \frac{a}{\sqrt{2}}$ it is maximum.



5.] An infinite plane sheet of charge (non-conducting)

By Gauss's theorem $\oint \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0}$

$$\int_1 \vec{E} \cdot d\vec{s} + \int_2 \vec{E} \cdot d\vec{s} + \int_3 \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0}$$

$$\int_1 E ds + \int_2 E ds + 0 = \frac{q}{\epsilon_0}$$

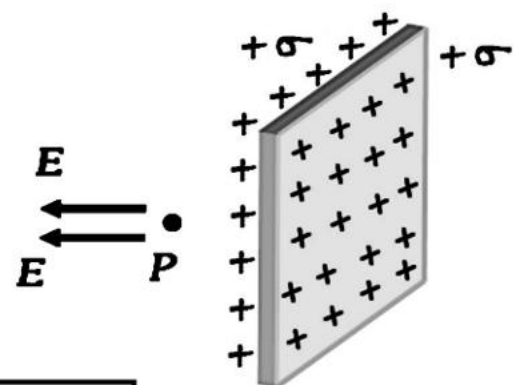
$$E \int_1 ds + E \int_2 ds = \frac{q}{\epsilon_0}$$

$$EA + EA = \frac{q}{\epsilon_0} = \frac{\sigma A}{\epsilon_0} \Rightarrow$$

$$E = \frac{\sigma}{2\epsilon_0}$$

6.] An infinite conducting plane sheet of charge

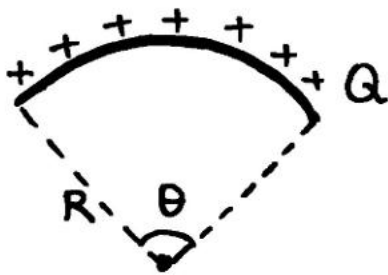
When a charge is given to a conducting plate, it distributes itself over the entire surface of the plate. The surface density σ is uniform and is the same on both surfaces.



Hence $E = 2 \times \frac{\sigma}{2\epsilon_0} \Rightarrow$

$$E = \frac{\sigma}{\epsilon_0}$$

Electric field due to charged circular Arc



$$E = \frac{2K\lambda}{R} \sin\left(\frac{\theta}{2}\right)$$

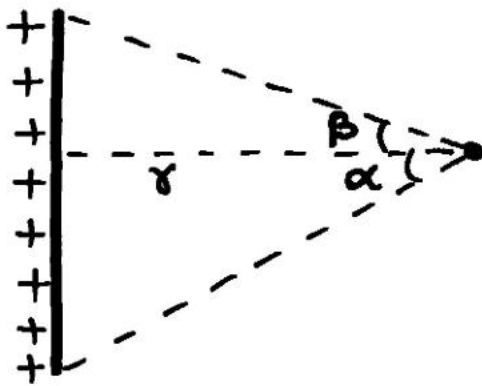
(i) If $\theta = \pi$ (Half Ring)

$$E = \frac{2K\lambda}{R}$$

(ii) If $\theta = 2\pi$ (Full Ring)

$$E = 0$$

Electric field due to Line charge



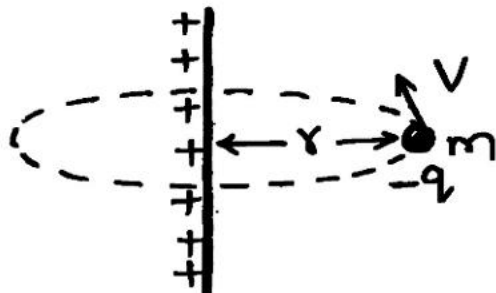
$$\vec{E} = \frac{K\lambda}{r} \left[(\sin\alpha + \sin\beta)\hat{i} + (\cos\alpha - \cos\beta)\hat{j} \right]$$

Special case -

If $\alpha = \beta$ then

$$\vec{E} = \frac{2K\lambda}{r} \sin\alpha \hat{i}$$

Charge moving around infinite line charge



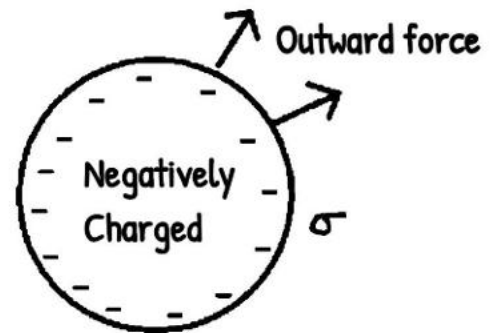
$$\frac{2K\lambda q}{r} = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{2K\lambda q}{m}}$$

Equilibrium of charged Soap Bubble

The air pressure inside the soap bubble is $4T/r$ more than the atmospheric pressure.

Pressure excess $p = \frac{4T}{r}$



Now if the surface charge density on charging the soap bubble is σ then electric pressure on this charged bubble towards the outer side of the surface will be $\frac{\sigma^2}{2\epsilon_0}$.

When the bubble is charged then a condition comes when the pressure excess becomes zero. In this condition the charged bubble bursts.

Now in equilibrium state $\frac{4T}{r} = \frac{\sigma^2}{2\epsilon_0}$

Radius of the bubble

$$r = \frac{8T\epsilon_0}{\sigma^2}$$

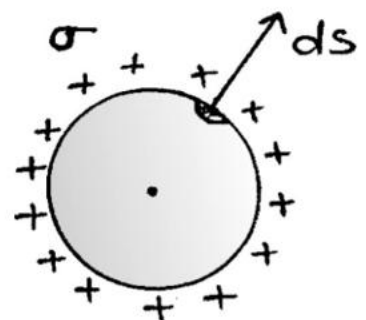
Electrostatic Pressure

The electric force acting per unit area of charged surface is defined as electrostatic pressure.

Force on small element ds of charged Conductor

$$dF = (\text{charge on } ds) \times \text{Electric field}$$

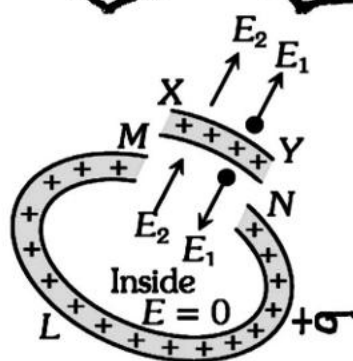
$$dF = (\sigma ds) \times \frac{\sigma}{2\epsilon_0} \Rightarrow dF = \frac{\sigma^2}{2\epsilon_0} ds$$



Hence $P_{ele.} = \frac{dF}{ds} = \frac{\sigma^2}{2\epsilon_0}$ (Normally Outwards)

Inside $E_1 - E_2 = 0 \Rightarrow E_1 = E_2$
 Outside $E_1 + E_2 = E \Rightarrow E = 2E_2$

$$E_2 = \frac{E}{2} = \frac{\sigma}{2\epsilon_0}$$



- Q. A large plane sheet of charge having surface charge density $5 \times 10^{-6} \text{ C/m}^2$ lie in xy plane. Find the electric flux through a circular area of radius 0.1 m if the normal to the circular area makes an angle of 60° with the z -axis.

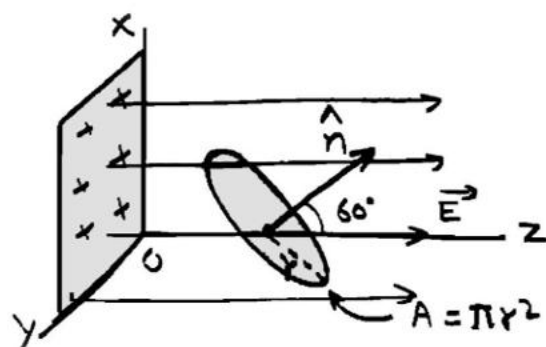
Sol. Here $\sigma = 5 \times 10^{-6} \text{ C/m}^2$
 $r = 0.1 \text{ m}$, $\theta = 60^\circ$

$$\text{So } \phi = \vec{E} \cdot d\vec{s} = E ds \cos \theta$$

$$\phi = \left(\frac{\sigma}{2\epsilon_0} \right) \times \pi r^2 \cos \theta$$

$$\phi = \frac{5 \times 10^{-6}}{2 \times 8.85 \times 10^{-12}} \times 3.14 \times (0.1)^2 \cos 60^\circ$$

$$\phi = 4.44 \times 10^3 \text{ Nm}^2/\text{C}$$



- Q. A charge of $17.7 \times 10^{-4} \text{ C}$ is distributed over a large sheet of area 200 m^2 . Calculate the electric field at a distance of 20 cm from it.

Sol. Surface charge density $\sigma = \frac{Q}{A}$

$$\sigma = \frac{17.7 \times 10^{-4}}{200} = 8.85 \times 10^{-6} \text{ C/m}^2$$

$$\text{Electric field } E = \frac{\sigma}{2\epsilon_0}$$

$$E = \frac{8.85 \times 10^{-6}}{2 \times 8.85 \times 10^{-12}} = 5 \times 10^5 \text{ N/C}$$

Q. At a point 20 cm from the centre of a charged non conducting sphere of radius 10 cm, the electric field is 100 V/m. Find the electric field at 3 cm from the centre of the sphere.

Sol. Electric field outside the sphere $E_{out} = \frac{kQ}{r^2}$
 Electric field inside the sphere $E_{in} = \frac{kQx}{R^3}$

$$\therefore \frac{E_{in}}{E_{out}} = \frac{r^2 x}{R^3}$$

$$\begin{aligned} E_{in} &= \frac{r^2 x}{R^3} \times E_{out} \\ &= \frac{3 \times (20 \times 10^{-2})^2}{(10 \times 10^{-2})^3} \times 100 = 120 \text{ V/m} \end{aligned}$$